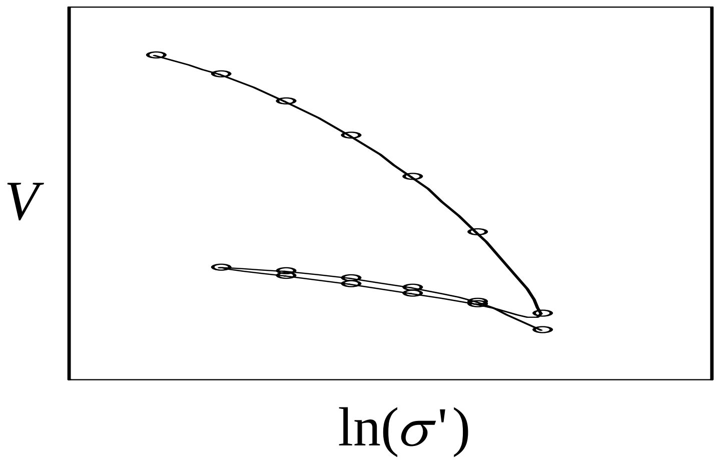
ELASTOPLASTICIDAD

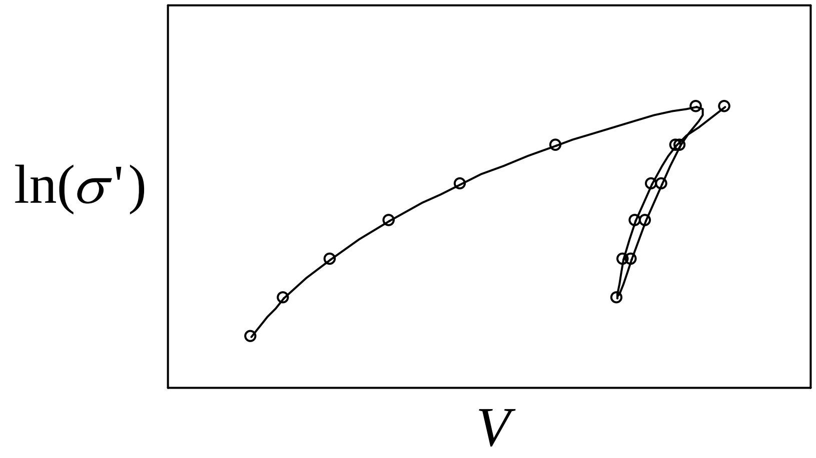
MODELO CAM CLAY

UNIVERSIDAD DE LOS ANDES

GRUPO DE INVESTIGACIÓN GEOTECNIA

ELASTOPLASTICIDAD





$$\partial \varepsilon = \partial \varepsilon^{e} + \partial \varepsilon^{p}$$

$$\partial \varepsilon_p^e + \partial \varepsilon_q^e$$

$$\partial \varepsilon_p^p + \partial \varepsilon_q^p$$

Deformaciones elásticas:

$$\begin{bmatrix} \partial\varepsilon_a \\ \partial\varepsilon_r \end{bmatrix} = \frac{1}{E'}\begin{bmatrix} 1 & -2\nu' \\ -\nu' & 1-\nu' \end{bmatrix} \cdot \begin{bmatrix} \partial\sigma_a' \\ \partial\sigma_r' \end{bmatrix}$$

$$\partial\varepsilon_a = \frac{1}{E}\left(\partial\sigma_a' - 2\nu\partial\sigma_r'\right)$$

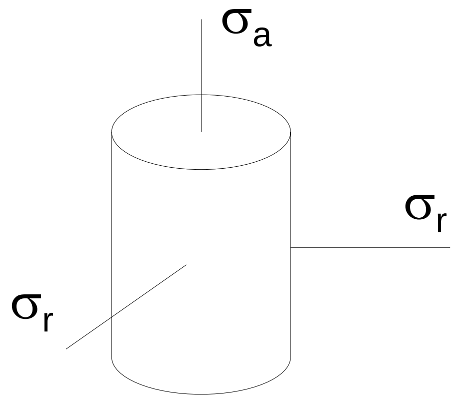
$$\partial\varepsilon_r = \frac{1}{E}\left(-\nu\partial\sigma_a' + (1-\nu)\partial\sigma_r'\right) \qquad (1)$$

$$\begin{bmatrix} p' \\ q \end{bmatrix} = \begin{bmatrix} 1/3 & 2/3 \\ 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} \sigma_a' \\ \sigma_r' \end{bmatrix}$$

$$\sigma_a' = p' + \frac{2q}{3}$$

$$\sigma_r' = p' - \frac{q}{3} \qquad (2)$$

$$\begin{bmatrix} \partial\varepsilon_p^e \\ \partial\varepsilon_q^e \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2/3 & -2/3 \end{bmatrix} \cdot \begin{bmatrix} \partial\varepsilon_a \\ \partial\varepsilon_r \end{bmatrix} \qquad (3)$$



uno y dos en tres se obtiene:

$$\begin{bmatrix} \partial\varepsilon_p^e \\ \partial\varepsilon_q^e \end{bmatrix} = \frac{1}{E}\begin{bmatrix} 3\partial p' - 6\nu\partial p' \\ \frac{2\nu}{3}\partial q + \frac{2\partial q}{3} \end{bmatrix} = \begin{bmatrix} \frac{3-6\nu}{E}\partial p' \\ \frac{2+2\nu}{3E}\partial q \end{bmatrix}$$

$$\begin{bmatrix} \partial\varepsilon_p^e \\ \partial\varepsilon_q^e \end{bmatrix} = \begin{bmatrix} \dfrac{3-6\nu}{E}\partial p' \\ \dfrac{2+2\nu}{3E}\partial q \end{bmatrix}$$

$$\begin{bmatrix} \partial\varepsilon_p^e \\ \partial\varepsilon_q^e \end{bmatrix} = \begin{bmatrix} \dfrac{3(1-2\nu)}{E}\partial p' \\ \dfrac{2(1+\nu)}{3E}\partial q \end{bmatrix},$$

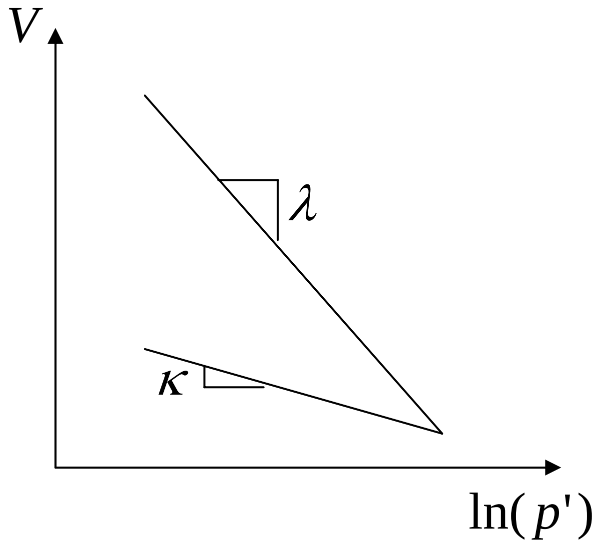
$$K' = \frac{E}{3(1-2\nu)}$$

$$G' = \frac{E}{2(1+\nu)}$$

$$\begin{bmatrix} \partial\varepsilon_p^e \\ \partial\varepsilon_q^e \end{bmatrix} = \begin{bmatrix} 1/K' & 0 \\ 0 & 1/3G' \end{bmatrix} \cdot \begin{bmatrix} \partial p' \\ \partial q \end{bmatrix}$$

$$\partial\varepsilon_p^e = \frac{1}{K'}\partial p'$$

$$\partial\varepsilon_q^e = \frac{\partial q}{3G'}$$



$$V = V_k - k \ln p' \qquad V - V_k = -k \ln p'$$

$$\partial V^e = -k \frac{\partial p'}{p'}$$

$$\partial \varepsilon_p^e = -\frac{\partial V^e}{V}$$

$$\partial \varepsilon_p^e = k \frac{\partial p'}{Vp'}$$

de la elasticidad se tenia $\partial \varepsilon_p^e = \frac{1}{K'} \partial p' \Rightarrow K' = \frac{Vp'}{k}$

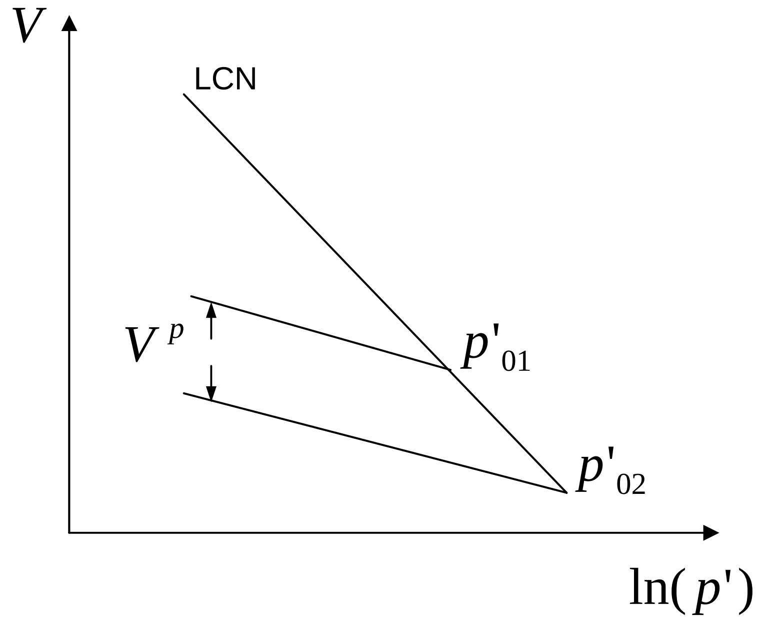
$$\partial \varepsilon_p^e = k \frac{\partial p'}{Vp'} \qquad \partial \varepsilon_q^e = \frac{\partial q}{3G'}$$

$$\partial \varepsilon = \partial \varepsilon^e + \partial \varepsilon^p$$

$$\partial \varepsilon^e = \partial \varepsilon_p^e + \partial \varepsilon_q^e$$

Deformaciones plásticas:

deformación volumétrica:



$$\Delta V = V^e + V^p$$

$$V = \lambda \ln p' + V_\lambda$$

$$V = k \ln p' + V_k$$

$$V^p = V_\lambda - V_k$$

$$V^p = -\lambda \ln\left(\frac{p'_{02}}{p'_{01}}\right) + k \ln\left(\frac{p'_{02}}{p'_{01}}\right)$$

$$V^p = -(\lambda - k) \ln\left(\frac{p'_{02}}{p'_{01}}\right)$$

$$V^p = -(\lambda - k)\ln\left(\frac{p'_{02}}{p'_{01}}\right)$$

$$\partial V^p = -(\lambda - k)\frac{\partial p'_0}{p'_0}$$

$$\partial \varepsilon_p^p = -\frac{\partial V^p}{V}$$

$$\partial \varepsilon_p^p = (\lambda - k)\frac{\partial p'_0}{V p'_0}$$

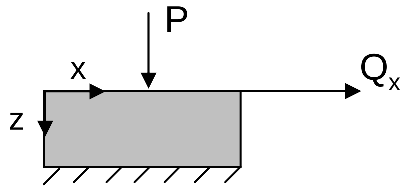
$$\partial \varepsilon = \partial \varepsilon^e + \partial \varepsilon^p$$

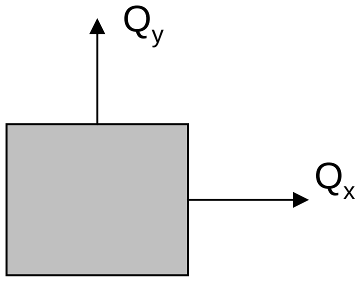
$$\partial \varepsilon^p = \partial \varepsilon_p^p + \partial \varepsilon_q^p$$

Deformación cortante plástica:

$$Qx = \mu P$$

$$f = Qx - \mu P = 0$$



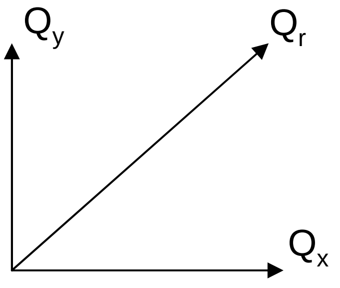


$$\sqrt{Qx^2 + Qy^2} = \mu P$$

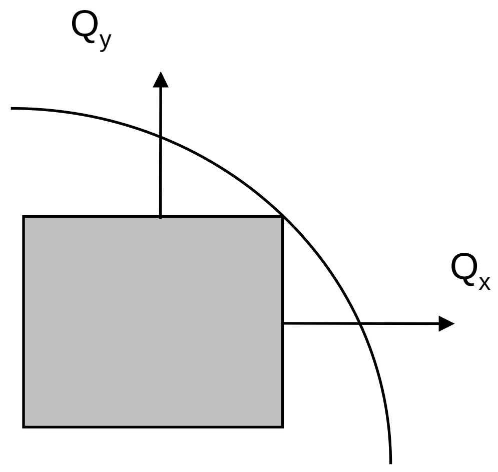
$$f = Qx^2 + Qy^2 - \mu^2 P^2 = 0$$

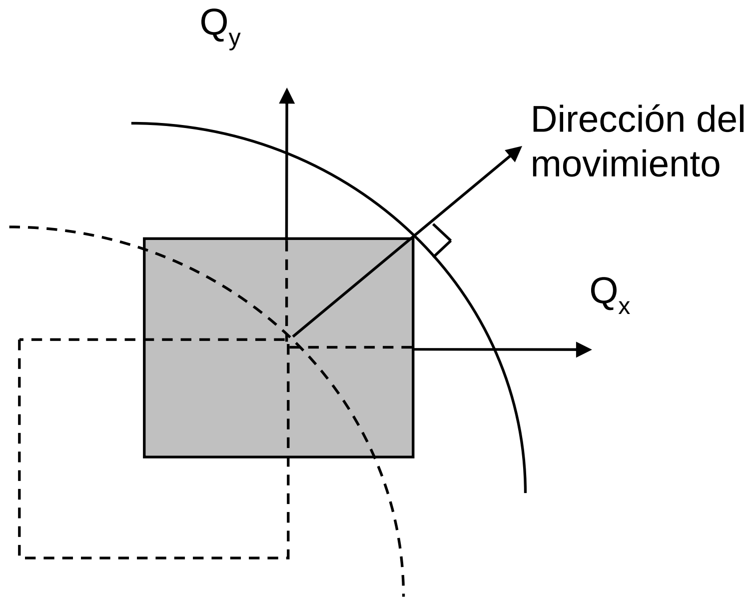
$$\partial x = \lambda \frac{\partial f}{\partial Qx} \qquad \partial \varepsilon_p^p = \lambda \frac{\partial f}{\partial p'}$$

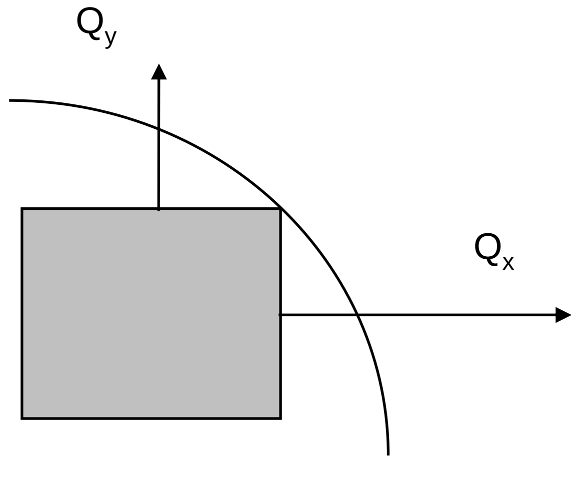
$$\partial y = \lambda \frac{\partial f}{\partial Qy} \qquad \partial \varepsilon_q^p = \lambda \frac{\partial f}{\partial q}$$

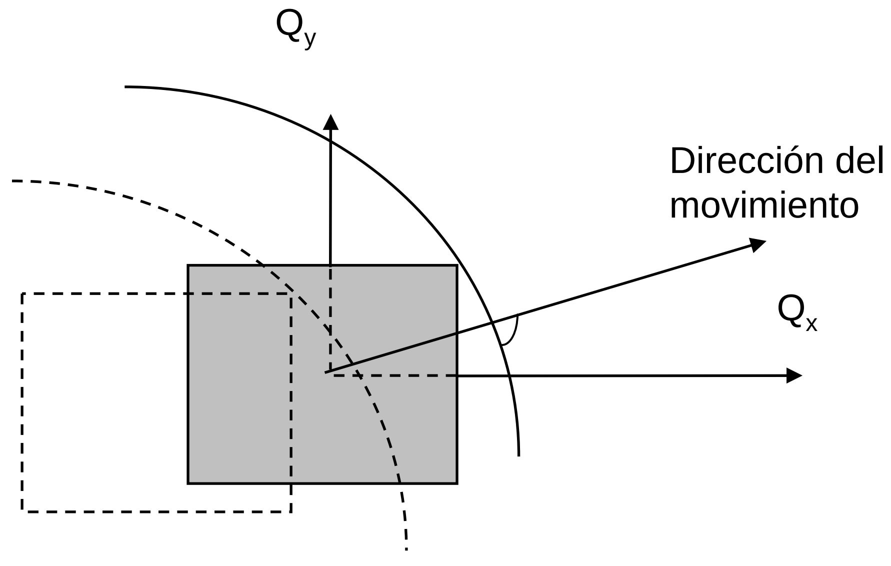


Superficie de fluencia y Potencial plástico

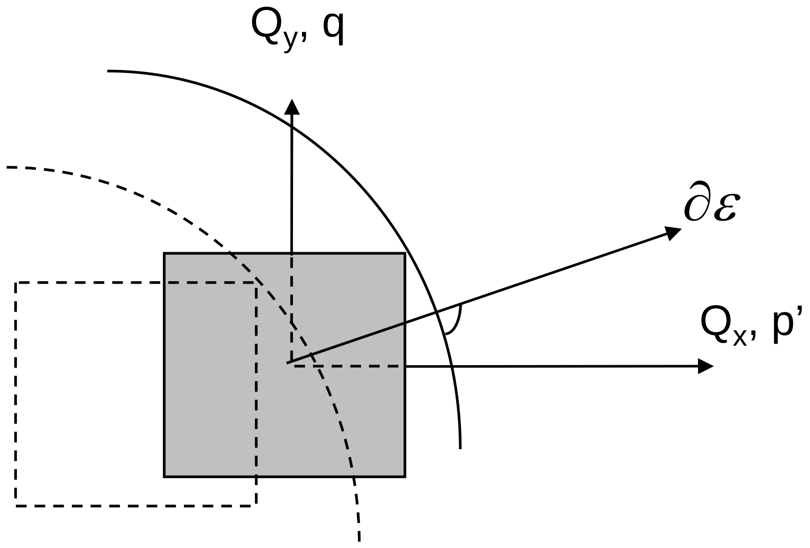


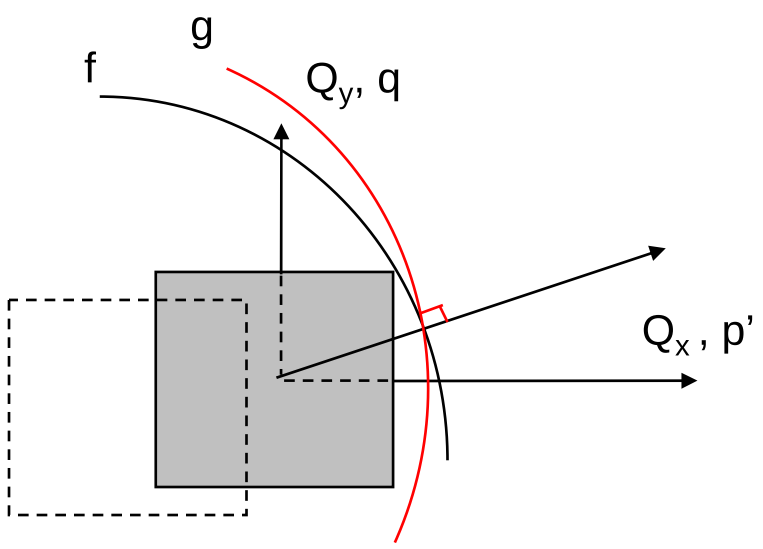






Potencial plástico y regla de flujo



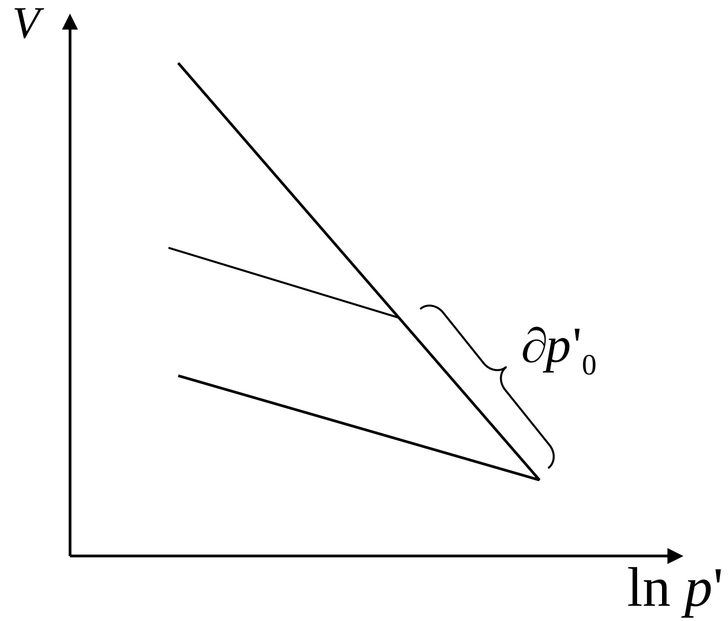


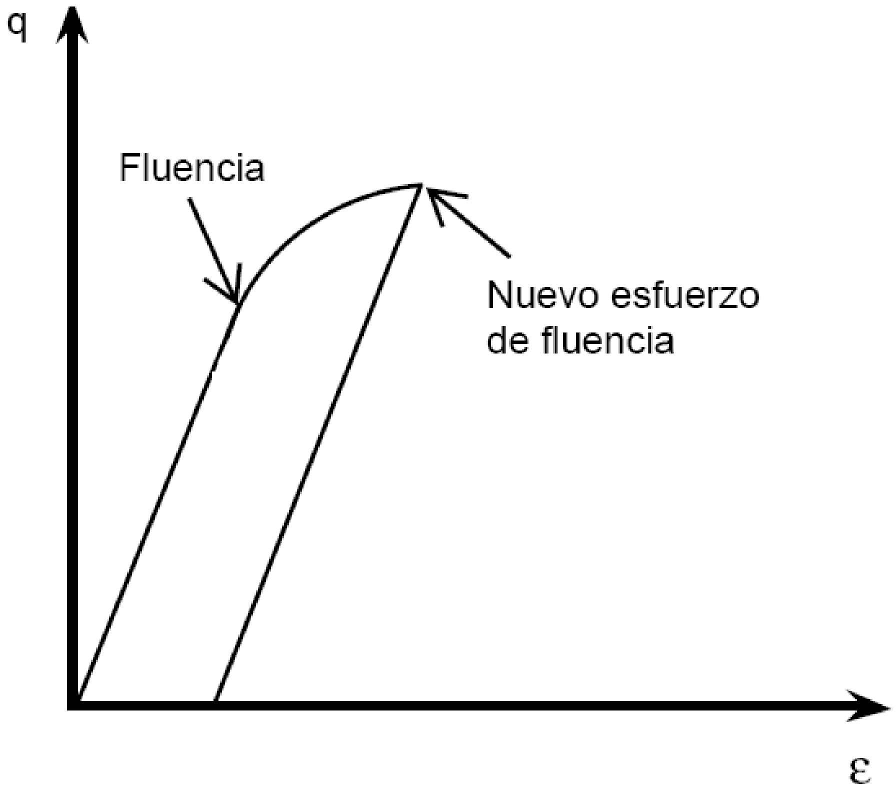
$$\partial \varepsilon_p^p = \lambda \frac{\partial g}{\partial p'}$$

$$\partial \varepsilon_q^p = \lambda \frac{\partial g}{\partial q}$$

$$\frac{\partial \varepsilon_p^p}{\partial \varepsilon_q^p} = \frac{\partial g / \partial p'}{\partial g / \partial q}$$

Regla de endurecimiento





$$\partial p'_0 = \frac{\partial p'_0}{\partial \varepsilon_p^p}\partial \varepsilon_p^p + \frac{\partial p'_0}{\partial \varepsilon_q^p}\partial \varepsilon_q^p$$

Resumen Elasto-plasticidad

Superficie de fluencia $f(p', q) = 0$

Potencial plástico $g(p', q) = 0$

Regla de flujo

$$\frac{\partial \varepsilon_p^p}{\partial \varepsilon_q^p} = \frac{\partial g / \partial p'}{\partial g / \partial q}$$

Regla de endurecimiento

$$\partial p'_0 = \frac{\partial p'_0}{\partial \varepsilon_p^p} \partial \varepsilon_p^p + \frac{\partial p'_0}{\partial \varepsilon_q^p} \partial \varepsilon_q^p$$

Modelo elástopástico Cam clay

Propiedades elásticas

Superficie de fluencia

Potencial plástico

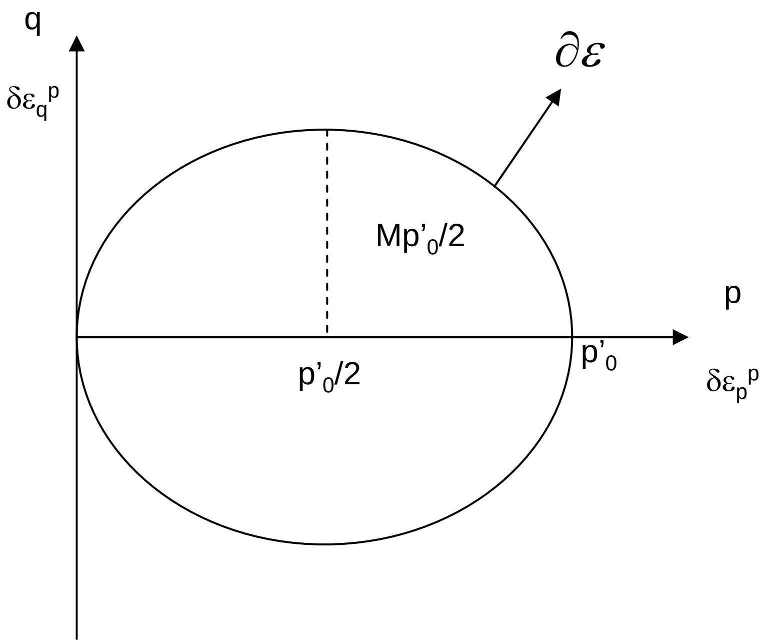
Regla de endurecimiento

$$\partial \varepsilon_p^e = k \frac{\partial p'}{Vp'}$$

$$\partial \varepsilon_q^e = \frac{\partial q}{3G'}$$

$$\partial \varepsilon_p^p = (\lambda - k) \frac{\partial p'_0}{Vp'_0}$$

$$\frac{\partial \varepsilon_p^p}{\partial \varepsilon_q^p} = \frac{\partial g / \partial p'}{\partial g / \partial q}$$



$$\frac{p'}{p'_0} = \frac{M^2}{M^2 + \eta^2}$$

$$\frac{p'\left(M^2+\eta^2\right) - M^2 p'_0}{p'_0\left(M^2+\eta^2\right)} = 0$$

$$p' M^2 + p'\eta^2 - M^2 p'_0 = 0$$

$$p' \frac{q^2}{p'^2} - M^2 p'_0 + p' M^2 = 0$$

$$\frac{q^2}{p'} - M^2\left(p'_0 - p'\right) = 0$$

$$f = q^2 - M^2\left[p'\left(p'_0 - p'\right)\right] = 0$$

$$g = f = q^2 - M^2\left[p'\left(p'_0 - p'\right)\right] = 0$$

$$\frac{\partial \varepsilon_p^p}{\partial \varepsilon_q^p} = \frac{\partial g / \partial p'}{\partial g / \partial q} = \frac{M^2 \left(2p' - p'_0\right)}{2q}$$

$$\frac{p'}{p'_0} = \frac{M^2}{M^2 + \eta^2} \Rightarrow p'_0 = \frac{p'\left(M^2 + \eta^2\right)}{M^2}$$

$$\frac{\partial \varepsilon_p^p}{\partial \varepsilon_q^p} = \frac{M^2\left(2p' - \frac{p'\left(M^2+\eta^2\right)}{M^2}\right)}{2q} = \frac{M^2\left(2p' - p' - \frac{p'\eta^2}{M^2}\right)}{2\eta p'} = \frac{2M^2 p' - M^2 p' - p'\eta^2}{2\eta p'}$$

$$\frac{\partial \varepsilon_p^p}{\partial \varepsilon_q^p} = \frac{M^2 - \eta^2}{2\eta}$$

Ecuaciones del modelo Cam Clay modificado

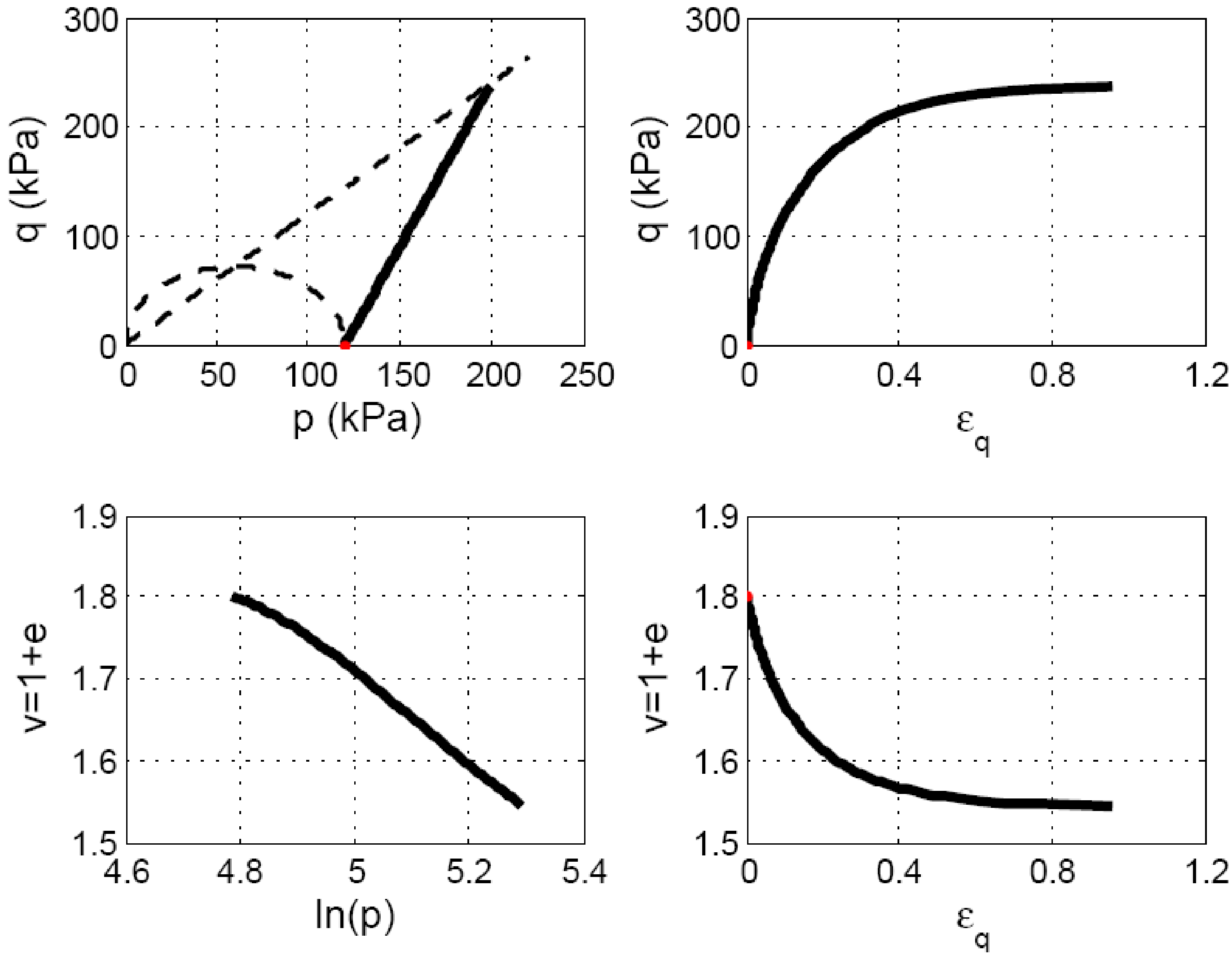
$$\partial \varepsilon_p^e = k \frac{\partial p'}{Vp'}$$

$$\partial \varepsilon_q^e = \frac{\partial q}{3G'}$$

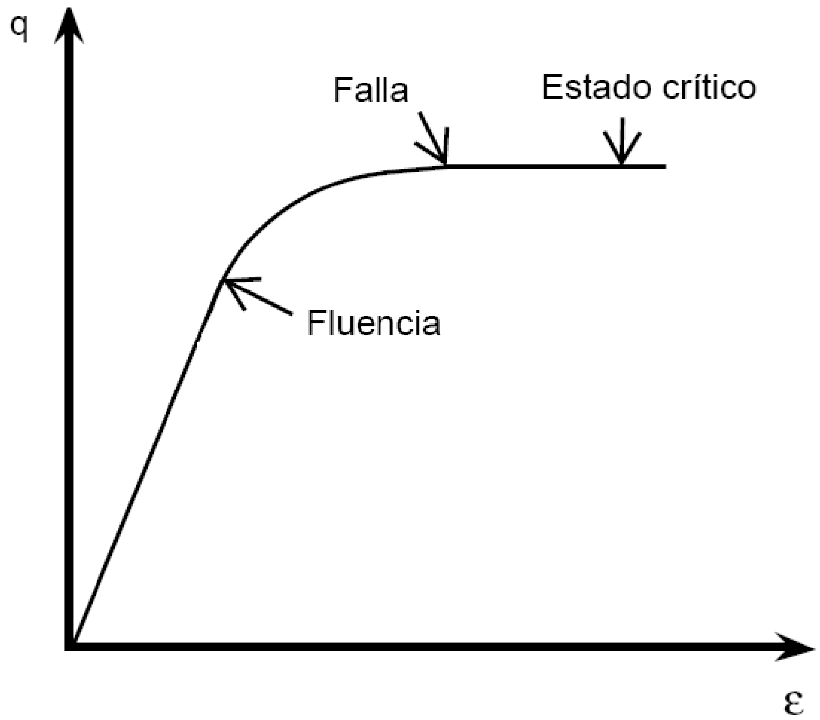
$$\partial \varepsilon_p^p = (\lambda - k) \frac{\partial p'_0}{Vp'_0}$$

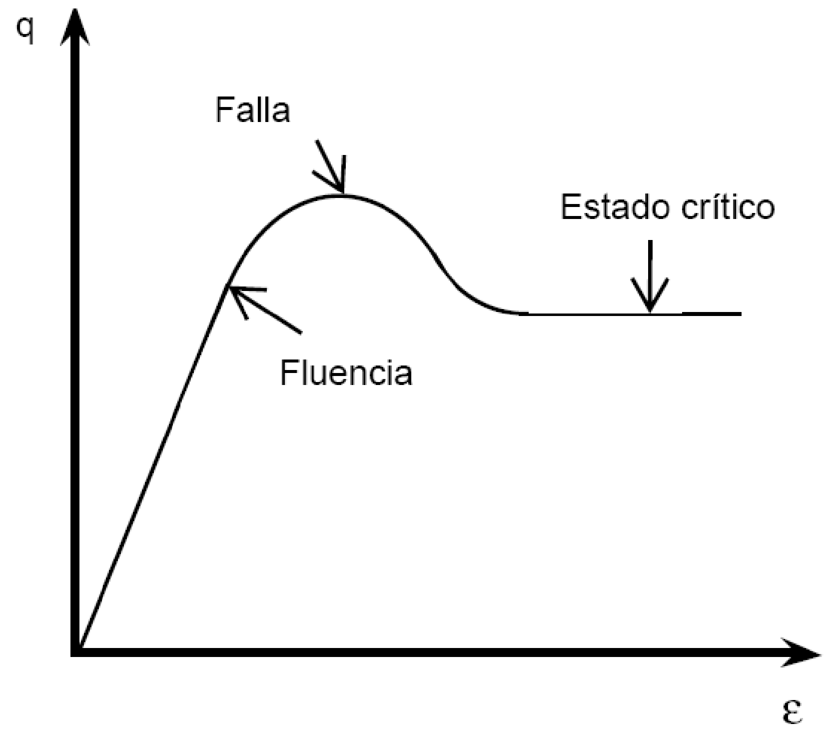
$$\frac{\partial \varepsilon_p^p}{\partial \varepsilon_q^p} = \frac{M^2 - \eta^2}{2\eta}$$

Resultados modelo Cam Clay modificado



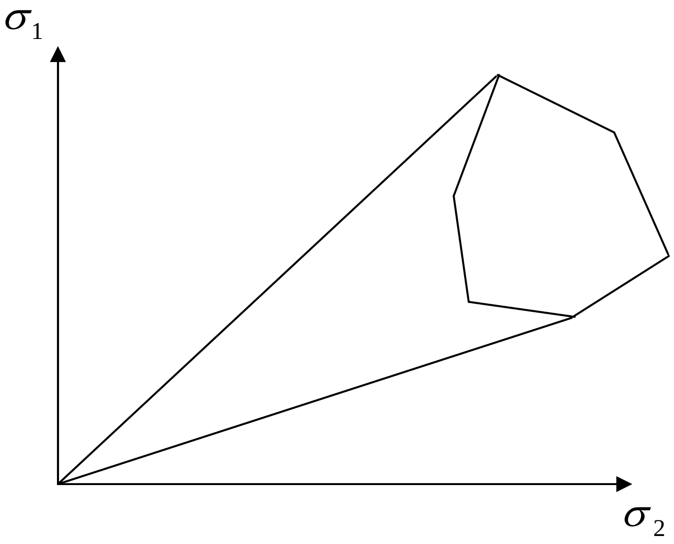
Fluencia, Falla y Estado Crítico

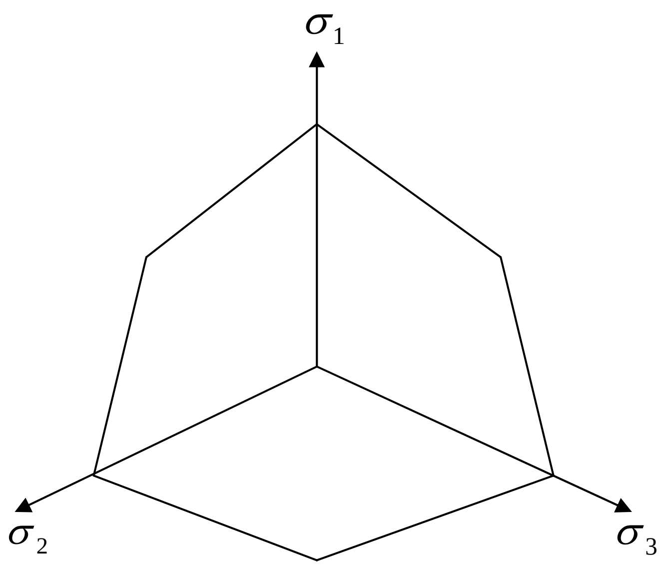




Criterios de falla

Mohr-Coulomb



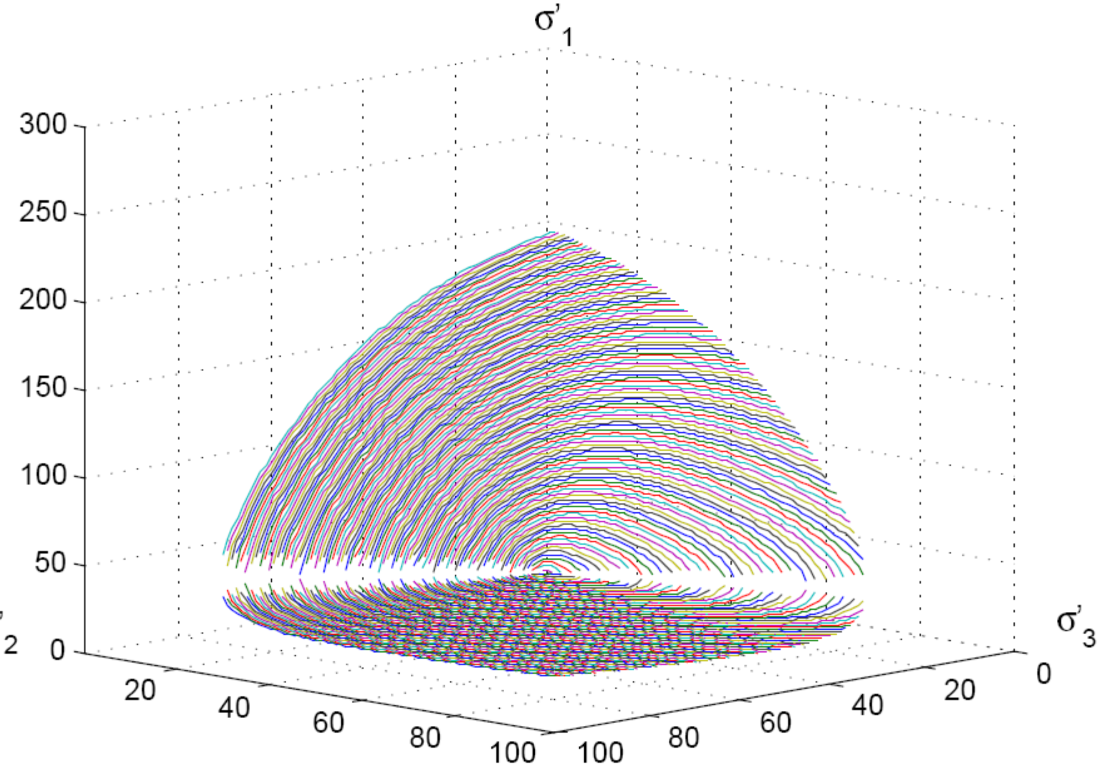


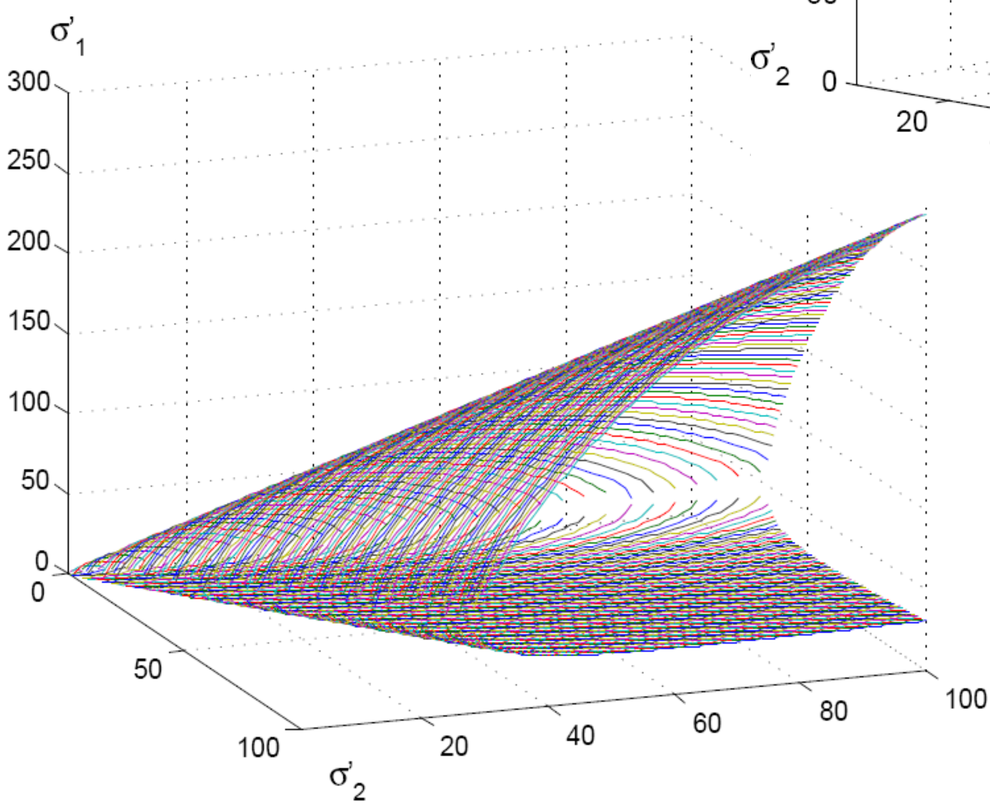
$$(\sigma_1 - \sigma_2) = 2c\cos\varphi + (\sigma_1 + \sigma_2)\sin\varphi$$

$$(\sigma_2 - \sigma_3) = 2c\cos\varphi + (\sigma_2 + \sigma_3)\sin\varphi$$

$$(\sigma_3 - \sigma_1) = 2c\cos\varphi + (\sigma_3 + \sigma_1)\sin\varphi$$

Matsuoka-Nakai



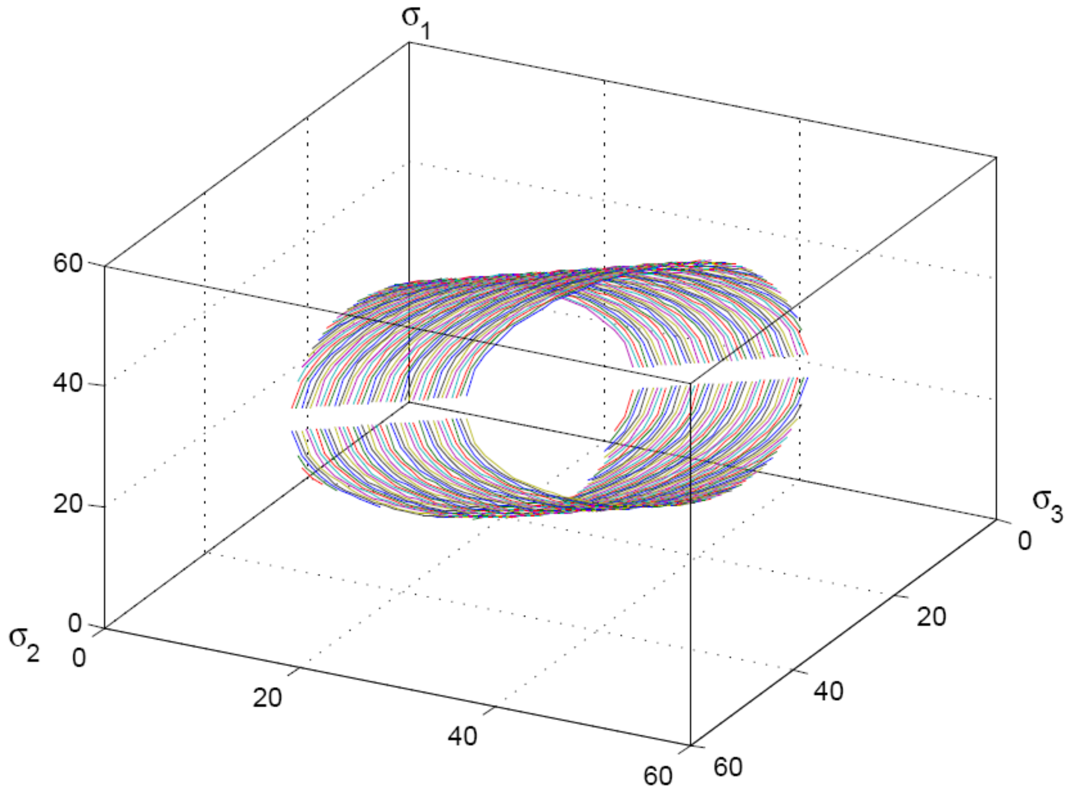


$$h_f = k_a^2 + \left(3 - \frac{k_a^2}{2}\right) - \left(\frac{q}{p'}\right)^2 = 0$$

$$k_a^2 = \frac{8 - \sin^2 \varphi}{3 + \sin^2 \varphi}$$

Von Mises

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = 2c^2$$



GRACIAS