A Visco-hypoplastic model for structured soils

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-CeiBA-

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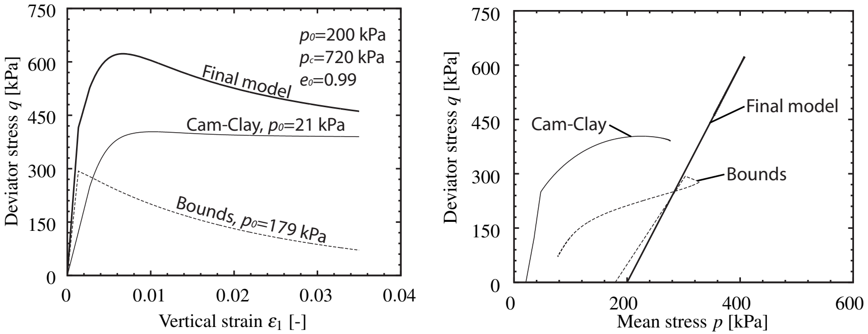
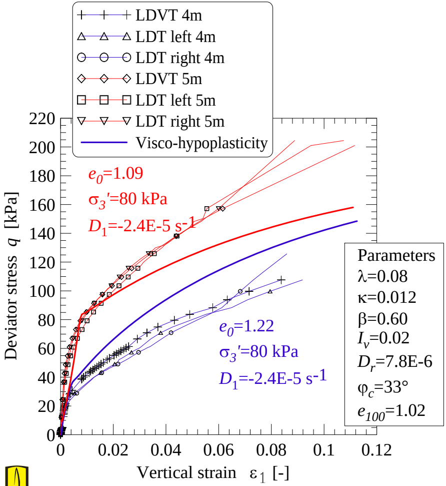
Motivation

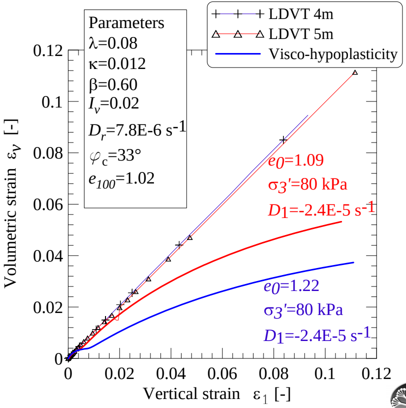
Nova, et.al model: drained triaxial test.



Motivation

- Drained triaxial test. Pampean loess sample.





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- Increase in the preconsolidation pressure

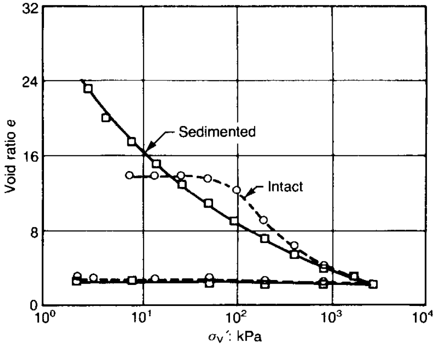
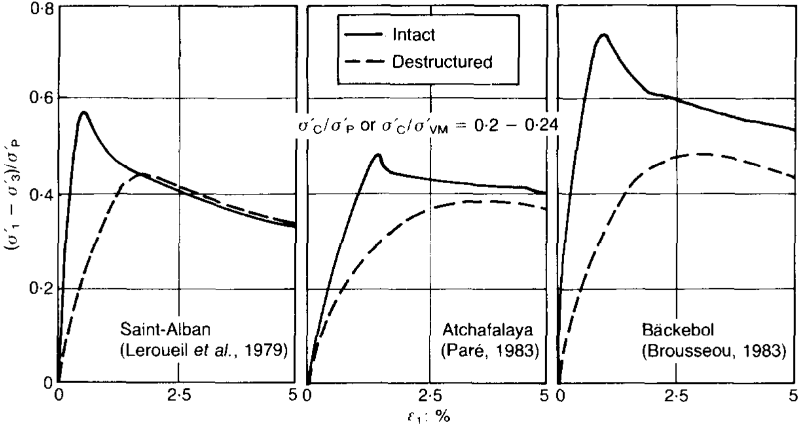


Figure: Oedometric test. Mexico city clay, Mesri (1975)

- Increase in strength and stiffness.



Other observations

- Viscous behaviour (Tatsuoka, et.al. 2002; Sorensen, et.al. 2007)
- Anisotropic yielding surface (Nova.et.al. 2001, Leroueil, et.al. 1990)

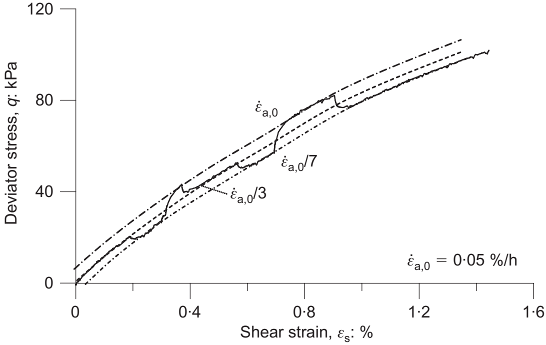


Figure: London clay isotachs, Sorensen, et.al. 2007

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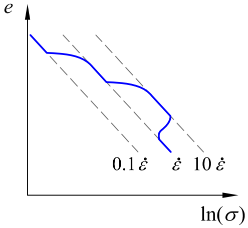
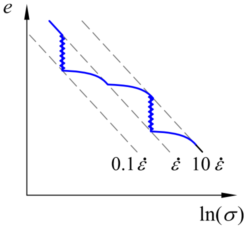
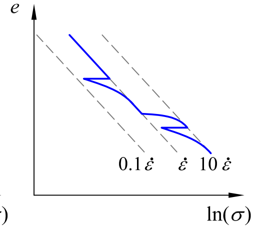
[1]

Assumptions:

- Decomposition of strains $\mathbf{D} = \mathbf{D}^{\mathbf{v}} + \mathbf{D}^{\mathbf{e}}$
- Simulates viscous behaviour, $h(\mathbf{T}, \lambda\mathbf{D}) \neq \lambda h(\mathbf{T}, \mathbf{D}) \neq \lambda \overset{\circ}{\mathbf{T}}$.
- Creep, stress relaxation and deformation rate dependence.
- BUTTERFIELD (1979) compression law. $\varepsilon = -\ln\left[\frac{1+e}{1+e_0}\right] = \lambda \ln\left[\frac{T}{T_0}\right]$.







[1] As reference model (Niemunis 1996, 2003).

Visco-hypoplasticity constitutive equation

$$\overset{\circ}{\mathbf{T}} = f_b \hat{\mathbb{L}} : (\mathbf{D} - \mathbf{D}^v) \tag{1}$$

where,

- $\overset{\circ}{\mathbf{T}}$ is the ZAREMBA-JAUMMAN objective stress rate tensor.
- $\mathbf{D}$ the strain rate tensor.
- $\mathbf{D}^v$ viscous strain rate tensor.

Fourth order **hypoelastic tensor** (Wolffersdorf, 1996):

$$\hat{\mathbb{L}} = a^2 \left(\left(\frac{F}{a} \right)^2 \mathbb{I} + \hat{\mathbf{T}} \otimes \hat{\mathbf{T}} \right) \tag{2}$$

$$\begin{cases} \hat{T} = \dfrac{\mathbf{T}}{\mathrm{Tr}\left[\mathbf{T}\right]}, & \text{The dimensionless stress tensor;} \\ f_b, & \text{The barotropy factor;} \\ F, a, & \text{Scalar functions which represent the limit surface;} \\ \mathbb{I}_{ijkl} = \dfrac{1}{2}(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}), & \text{The fourth order unit tensor for symmetric tensors.} \end{cases} \tag{3}$$

Barotropy factor

$$f_b = -\frac{\mathrm{tr}\mathbf{T}}{(1 + a^2/3)\kappa} = -\beta_b \mathrm{tr}\mathbf{T} \tag{4}$$

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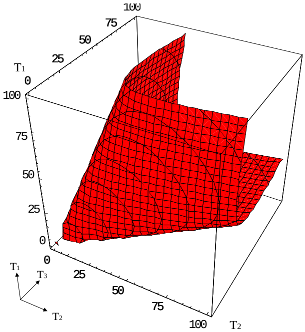
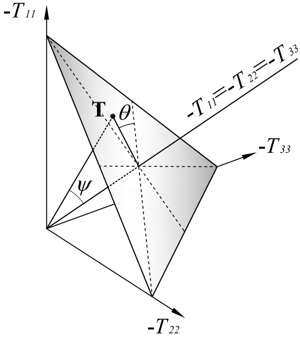
Critical state surface

$$\hat{\mathbb{L}} = a^2\left(\left(\frac{F}{a}\right)^2 \mathbb{I} + \hat{\mathbf{T}}\otimes\hat{\mathbf{T}}\right)$$ (Wolffersdorff, 1996).





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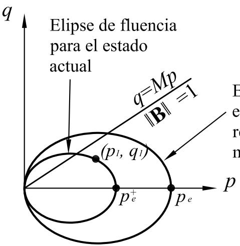
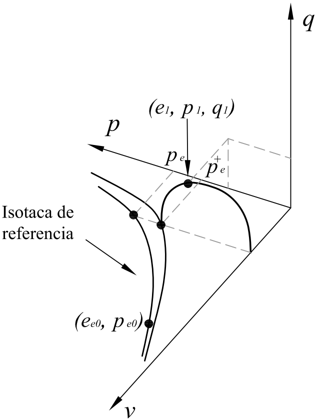
3D definition for OCR: CAM-CLAY surface.





OCR definition

$$\mathrm{OCR} = \frac{p_e}{p_e^+} \tag{13}$$

Parameters

Name	Symbol	Units	Test
Compression index	λ	[-]	Oedometric
Swelling index	κ	[-]	Oedometric
Critical friction angle	φ_c	$[^\circ]$	Triaxial
Viscosity index	I_v	[-]	Oedometric
Reference creep rate	D_r	[-]	Oedometric
Void ratio for 100 kPa	e_{100}	[-]	Oedometric

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Review of some models:

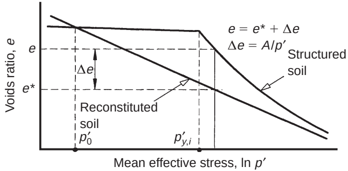
Compression law

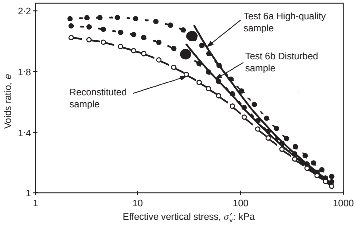
Liu and Carter 1999

$$\Delta e = S\left(\frac{p_{c0}}{p}\right)\ln(p_{c0}) \qquad (17)$$

Other proposal:
Masin (2006)

$$\Delta \ln(1+e) = \lambda \ln(s) \qquad (18)$$





Model from Liu and Carter (1999)

Review of some models:

Incorporation of a structure variable

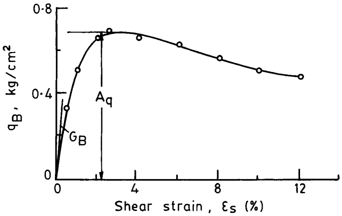
Decomposition of stresses; Nova et.al. (2001), Bauer y Wu (1993)

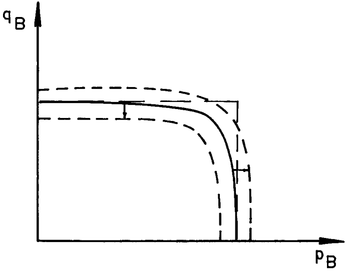
$$\mathbf{T} := \mathbf{T}_R + \mathbf{T}_B \qquad (19)$$

Increase in the preconsolidation stress Stallebrass et.al. 2004, Masin, 2006:

$$p_e := s p_e \qquad (20)$$

Modification in the bounding surface: Wood et.al. (2000)





Model for bounded component Nova (1999)

Degradation law for stucture:

Nova, et.al model

$$f_B := \left(\frac{p_B}{A_p}\right)^m + \left(\frac{q_B}{A_q}\right)^m - N$$
$$dN := \frac{\partial N}{\partial p_B} K_B^p d\varepsilon_v^p - \alpha \frac{\partial N}{\partial q_B} q_B d\varepsilon_s^p \qquad (21)$$

Stallebrass, et.al model

$$\dot{s} := \frac{k}{\lambda - \kappa}(s - s_f)\dot{\varepsilon}^{dam}$$
$$\dot{\varepsilon}^{dam} := \sqrt{\dot{\varepsilon}_v^{p2} + \dot{\varepsilon}_s^{p2}} \qquad (22)$$

Masin model

$$\dot{s} := \frac{k}{\lambda - \kappa}(s - s_f)\dot{\varepsilon}^{dam}$$
$$\dot{\varepsilon}^{dam} := \sqrt{\dot{\varepsilon}_v^2 + \frac{A}{1 - A}\dot{\varepsilon}_s^2} \qquad (23)$$

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Proposed model

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First approximation

Modified HVORSLEV stress:

$$p_e^* = s p_e \tag{26}$$

Degradation law for structure[3]:

$$\dot{s} = -\frac{k}{\lambda}(s-1)\dot{\varepsilon}^{dam} \tag{27}$$

$$\dot{\varepsilon}^{dam} := \sqrt{\dot{\varepsilon}^v + \frac{A}{1-A}\dot{\varepsilon}^s} \tag{28}$$

with $\dot{\varepsilon}^v$ and $\dot{\varepsilon}^s$:

$$\dot{\varepsilon}^v = \mathrm{Tr}\left[\mathbf{D}\right] \qquad \dot{\varepsilon}^s = \sqrt{\frac{2}{3}} \parallel \mathbf{D}^* \parallel \tag{29}$$

and $\mathbf{D}^* = \mathbf{D} - \frac{1}{3}\dot{\varepsilon}^v\mathbf{1}$ is the deviatoric strain rate.

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[3] Adapted from (Stallebrass, et.al 2004)

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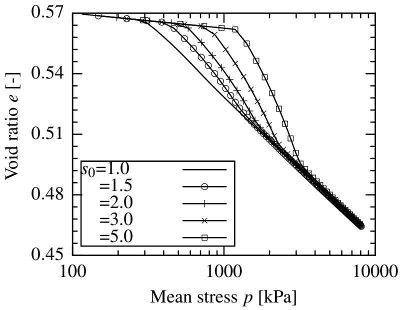

Parameters:

e_{100} $[-]$	λ $[-]$	κ $[-]$	β_R $[-]$	I_v $[-]$	D_r $[s^{-1}]$	φ_c $[^\circ]$	A $[-]$	k $[-]$
0.58	0.022	0.005	0.7	0.018	1.0e-6	22.3	0.5	0.3

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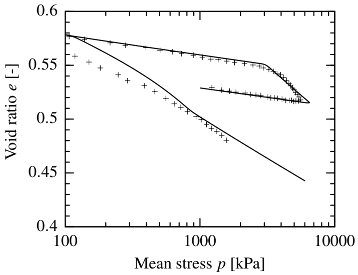
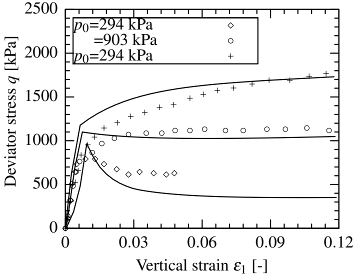
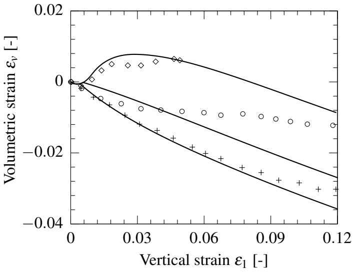
Simulations on Marl Clay







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New proposal

1. The new $\mathbf{D}^{vis}$ reads:

$$\mathbf{D}^{vis} := -D_r \hat{\mathbf{B}} \left(\frac{1}{OCR} \right)^{\frac{1}{Iv}} \left(\frac{1}{s} \right)^{\frac{1}{Iv}}$$

2. Where s follows the degradation law:

$$\dot{s} := -\frac{k}{\lambda}(s-1)\dot{\varepsilon}^{dam}$$

3. The damage function $\dot{\varepsilon}^{dam}$ is:

$$\dot{\varepsilon}^{dam} := \exp\left[-\omega \mathrm{OCR}^*\right] \sqrt{\dot{\varepsilon}^v + \dot{\varepsilon}^s}$$

4. The OCR^* is defined as:

$$\mathrm{OCR}^* = \left[\frac{\sqrt{3} D_r}{\| -\hat{\mathbf{B}}^{-1} \cdot \mathbf{D} \|} \right]^{-I_v} \mathrm{OCR}$$

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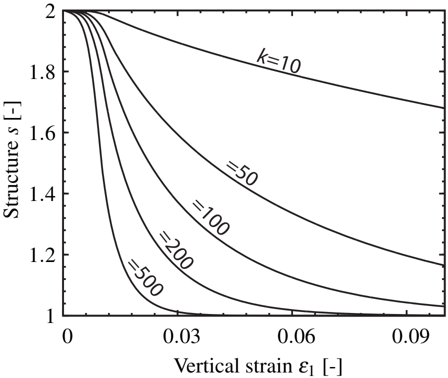
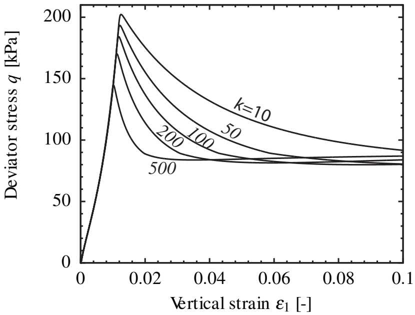
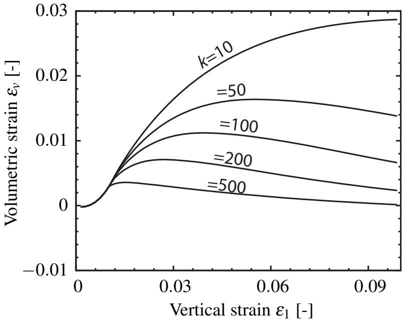
Drained triaxial test, $p_0 = 100$ kPa.

e_{100} [$-$]	λ [$-$]	κ [$-$]	β_R [$-$]	I_v [$-$]	D_r [s^{-1}]	φ_c [$^\circ$]	k [-]	Γ [-]	ω [-]	s_0 [-]
1.0	0.06	0.006	0.95	0.002	1.0e-5	18	-	0.05	1.0	2.0



Simulations





Factor of barotropy

LIU and CARTER compression law:

$$\Delta e = \Gamma \left(\frac{p_{y,i}}{p} \right) \ln p_{y,i} \qquad (39)$$

where,

- Γ the structure index (Parameter).
- $p_{y,i}$ the initial overconsolidated mean stress

Proposal:

$$\ln \left(\frac{1+e}{1+e_0} \right) = (-\kappa + \Gamma \ln(s)) \ln \left(\frac{p}{p_r} \right) \qquad (40)$$

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In rate form:

$$D^{vol} = (-\kappa + \Gamma \ln(s)) + \frac{\dot{s}}{s}\Gamma \ln(p) \tag{41}$$

For isotropic compression,

$$\dot{s} = \frac{k}{\lambda}(s-1)D^{vol} \tag{42}$$

The barotropy factor reads:

Factor of barotropy:

$$f_b := 3p\frac{1 - \frac{\Gamma}{s}\ln(p)\frac{k}{\lambda}(s-1)}{(\kappa - \Gamma \ln(s))\left(1 + \frac{a^2}{3}\right)} \tag{43}$$

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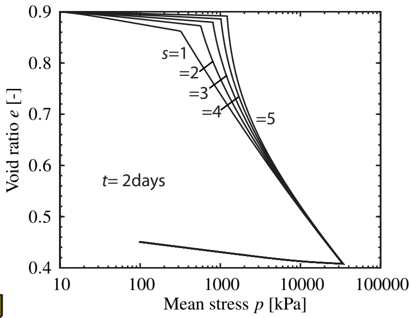
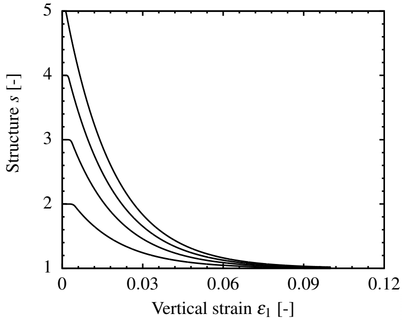
Simulations

Isotropic compression test.

e_{100}	λ	κ	β_R	I_v	D_r	φ_c	k	Γ	s_0
$[-]$	$[-]$	$[-]$	$[-]$	$[-]$	$[s^{-1}]$	$[^\circ]$	[-]	[-]	[-]
1.0	0.06	0.006	0.95	0.02	1.0e-5	18	5.0d1	6.0e-3	-





Hysteretical elasticity

The delayed deformation $\mathbf{h}$ is a state variable that follows,

$$\overset{\circ}{\mathbf{h}} = \left[\vec{\mathbf{D}} - \mathrm{Tanh}(c\mathbf{h}) \right] \parallel \mathbf{D} \parallel \tag{47}$$

where:

- $\overset{\circ}{\mathbf{h}}$ is the objective rate of delayed deformation,
- $\vec{\mathbf{D}} = \mathbf{D} / \parallel \mathbf{D} \parallel$ is the unit rate of deformation, and
- c is a material parameter.

The rigidity factor f_r is defined as:

$$f_r := A \left(1 - \exp \left(- \frac{\parallel \dot{\mathbf{h}} \parallel}{\parallel \mathbf{D} \parallel} \right) \right)^{\chi} + 1 \tag{48}$$

χ and A are parameters, and $\mathbb{M} = f_r \hat{\mathbb{L}}$

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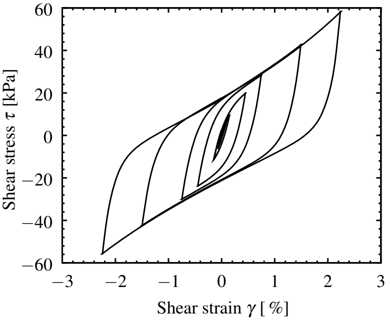
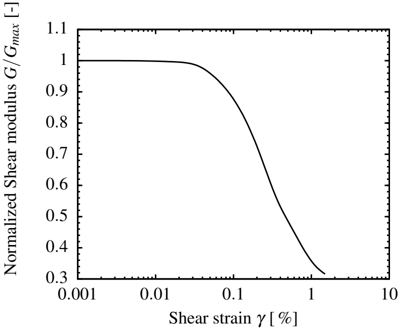
Ciclic triaxial test

Undrained ciclic triaxial test. $p_0 = 100kPa$

e_{100}	λ	κ	β_R	I_v	D_r	φ_c	c	A	χ	k	Γ	s_0
$[-]$	$[-]$	$[-]$	$[-]$	$[-]$	$[s^{-1}]$	$[^\circ]$	[-]	[-]	[-]	[-]	[-]	[-]
1.5	0.2	0.025	0.95	0.02	1.0e-5	18	4.0d2	20.0	2.0	2.0d2	3.0e-3	2

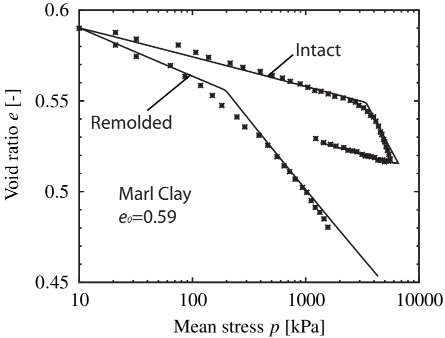




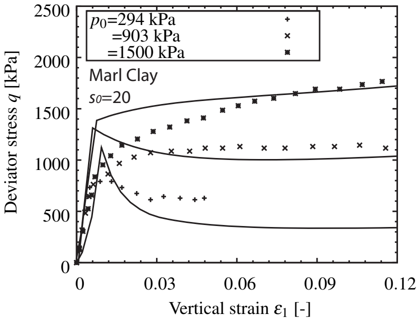
Simulations on Marl Clay

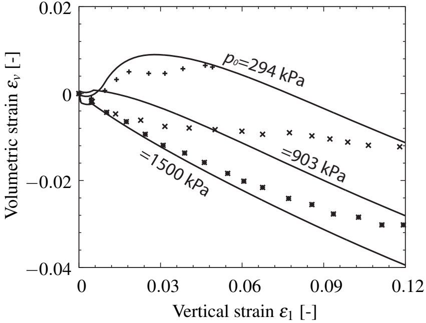
Oedometric test.

e_{100}	λ	κ	β_R	I_v	D_r	φ_c	k	Γ	ω	s_0
$[-]$	$[-]$	$[-]$	$[-]$	$[-]$	$[s^{-1}]$	$[^\circ]$	[-]	[-]	[-]	[-]
0.58	0.022	0.0073	0.7	0.018	1.0e-6	18	1.7d1	1.0d-3	8d-3	20



Drained triaxial test, $s_0 = 20$





Proposed model

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Conclusions

- An extension for structured soils has been developed in the hypoplasticity framework.
- The model reproduces viscous effects, and the degradation of structure.
- Structure state variables are defined in a current configuration as the reference configuration.
- It reproduces well the Marl Clay characterized for its high cementation.
- A FEM user routine has been developed.

Limitations

- Does not simulates anisotropy.
- Supposes a unique critical state for remolded an intact material.
- Unaceptable responses envelopes.

Thanks

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