Bifurcation theory applied to strain localization

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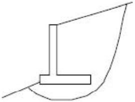
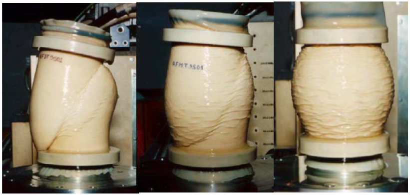
Physical phenomenon in practice of geotechnical engineering. Monotonic and cyclic loading


Dalhaus. 2001

http://gees.usc.edu/GEER/

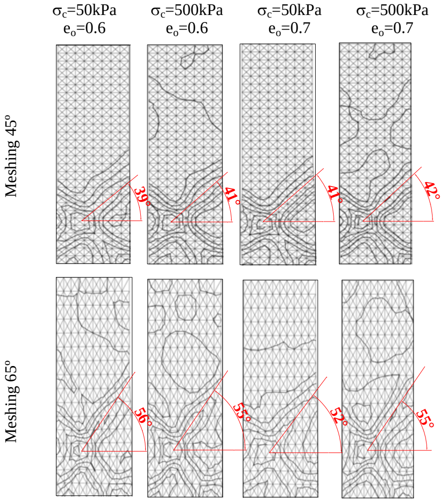


Desrue. 2004

Desrues, 2002

Lizcano et al., 1998

Inclination of shear band



Shortcomings of FEM to simulate strain localization

- Calibration of constitutive models. Both, field of stress and strain have to be homogeneous.
- When it is simulated a geotechnical structure near to peak stress or softening branch, it is attempt to simulating a phenomena of discontinuity strain by mean a theory based on the mechanics of continuum media.
- Conventional continuum does not have a characteristic length associated that permits define a localization band to regularize the problem. For that reason, the results are not unique depending on discretization used in the FEM analysis.

Since those shortcomings it is relevant to know until when it is valid the constitutive modelling either elemental test level or FEM level in a BVP.

Content

Hypoplastic Model (Wolffersdorff, 1996)

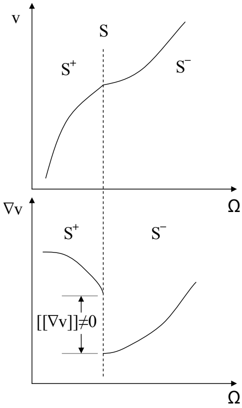
$$\dot{\sigma} = \mathbf{C}^{hyp} : \dot{\varepsilon}$$

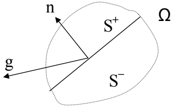
$$\overset{\circ}{\mathbf{T}} = L : \mathbf{D} + \mathbf{N}\|\mathbf{D}\|$$

$$\overset{\circ}{\mathbf{T}} = \underbrace{\left[L + \mathbf{N}\frac{\mathbf{D}}{\|\mathbf{D}\|} \right]}_{\mathbf{C}^{hyp}} : \mathbf{D}$$

Test	Parameter
Repose angle	φ_c
e_{max}, e_{min}	e_{io}, e_{co}, e_{do}
Oedometric compression	h_s, n, β
Drained triaxial	α

A specimen initially homogeneous is strained up to a state $(\mathbf{T}, e)$. **Question**: Besides a homogeneous deformation $(\mathbf{v}, \nabla\mathbf{v})$, the equations permit a nonhomogeneous strain in a discontinuity plane S





$$[[\nabla \mathbf{v}]] = \nabla \mathbf{v}^{+} - \nabla \mathbf{v}^{-} \neq \mathbf{0} \quad [[\mathbf{v}]] = \mathbf{0}$$

Second Cauchy theorem

$$\dot{\mathbf{t}}^{+} = \dot{\mathbf{T}}^{+}\mathbf{n} \quad \dot{\mathbf{t}}^{-} = \dot{\mathbf{T}}^{-}\mathbf{n}$$

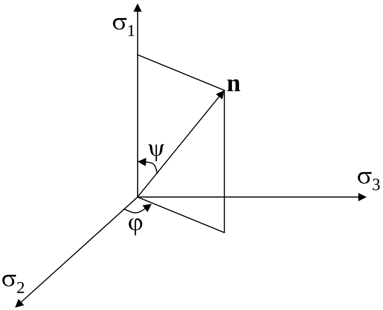
For equilibrium, the jump of the vector of stress rate across the discontinuity must be zero $[[\dot{\mathbf{t}}]] = \mathbf{0}$

$$[[\dot{\mathbf{T}}]]\mathbf{n} = \mathbf{0}$$

$$[[\dot{\mathbf{T}}]]\mathbf{n} = \left[[[\overset{\circ}{\mathbf{T}}]] + [[\mathbf{W}]]\mathbf{T} - \mathbf{T}[[\mathbf{W}]] \right] \mathbf{n} = \mathbf{0}$$

$$[[\nabla \mathbf{v}]] = \mathbf{g} \otimes \mathbf{n}$$
$$[[\mathbf{D}]] = \mathbf{D}^{+} - \mathbf{D}^{-} = [\mathbf{g} \otimes \mathbf{n} + \mathbf{n} \otimes \mathbf{g}]/2$$
$$[[\mathbf{W}]] = \mathbf{W}^{+} - \mathbf{W}^{-} = [\mathbf{g} \otimes \mathbf{n} - \mathbf{n} \otimes \mathbf{g}]/2$$
$$\gamma = \|\mathbf{D}^{+} - \mathbf{D}^{-}\| = \sqrt{[[\mathbf{D}]] : [[\mathbf{D}]]}$$

$$\mathbf{g} = \begin{bmatrix} g_1 \\ g_2 \\ g_3 \end{bmatrix} \quad \mathbf{n} = \begin{bmatrix} \cos\psi \\ \sin\psi\cos\varphi \\ \sin\psi\sin\varphi \end{bmatrix}$$



Direction for the weak discontinuity plane

$$\gamma = f(g_1, g_2, g_3, T, e, \psi, \varphi) = \sqrt{[[\mathbf{D}]] : [[\mathbf{D}]]} = 1$$

Finally, we have an equation for inclination of shear band.

$$f(\psi, \varphi) = 0$$

- At the beginning of the deformation process the equation does not have real roots.
- Depending on the conditions, It is possible to find a state (T, e) where the bifurcation conditions are fulfilled.

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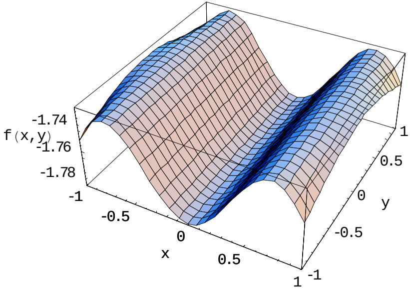
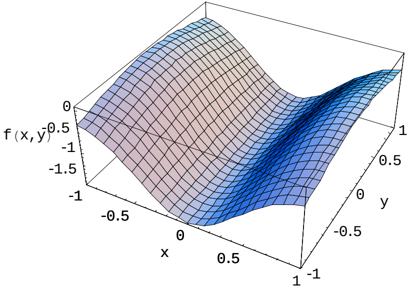
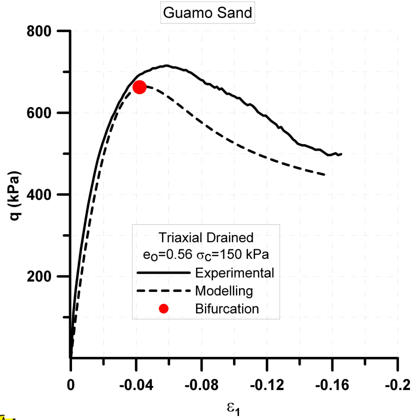
Function $f(\psi, \varphi)$ at the beginning of deformation

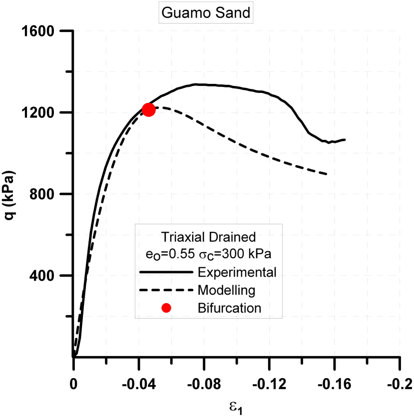


Function $f(\psi, \varphi)$ at onset of bifurcation



Bifurcation applied to triaxial - Guamo Sand hypoplastic constitutive model





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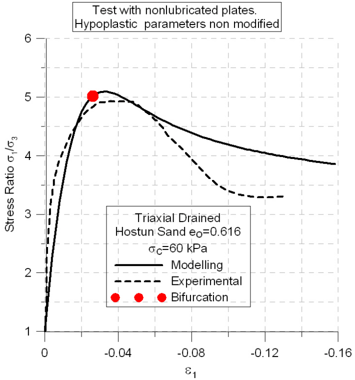
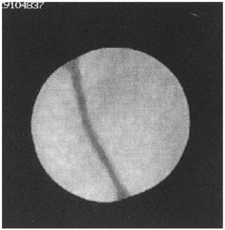
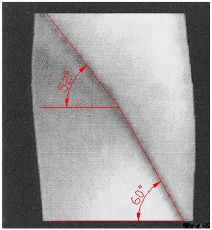
Bifurcation application to triaxial- Hostun sand



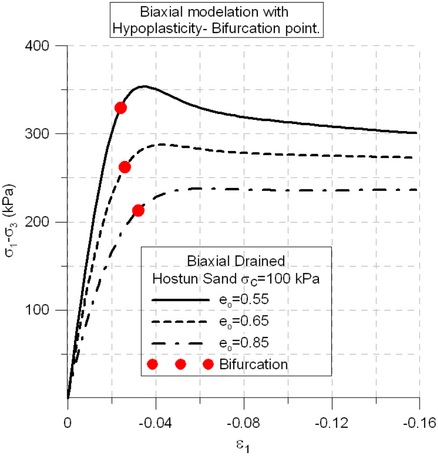




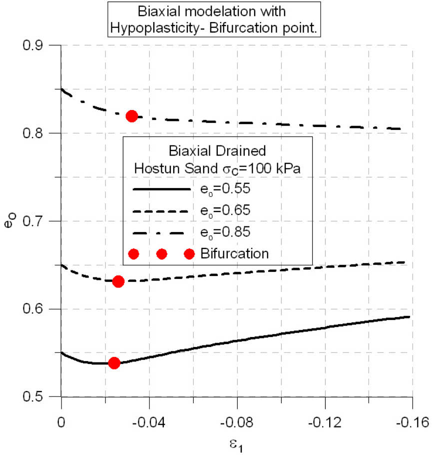
Hostun Sand Test Rftd4

Direction of shear band calculated by mean bifurcation = $62{,}35^o$

Application to Biaxial test. Hypoplasticity



Application to Biaxial test. Hypoplasticity

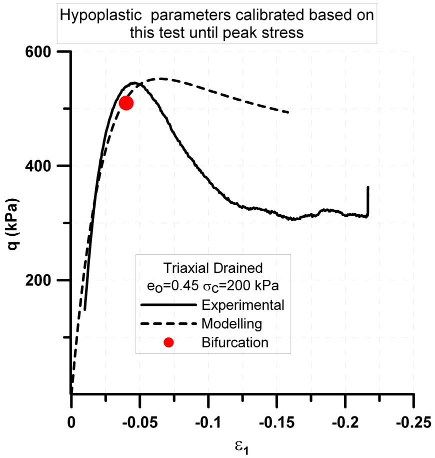


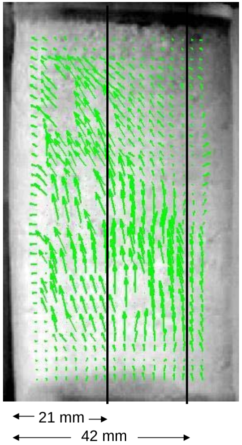
Sand	Karlsruhe	Hostun	Toyoura
$d_{50}[mm]$	0.40	0.35	0.16
e_o	0.62	0.62	0.69
$\theta^m_{80}[^o]$	59.0	63.0	66.0
$\theta^c_{80}[^o]$	60.21	62.69	61.7
e_o	0.63	0.65	0.66
$\theta^m_{400}[^o]$	58.0	58.0	66.0
$\theta^c_{400}[^o]$	59.3	60.1	61.1

80 kPa, 400 kPa

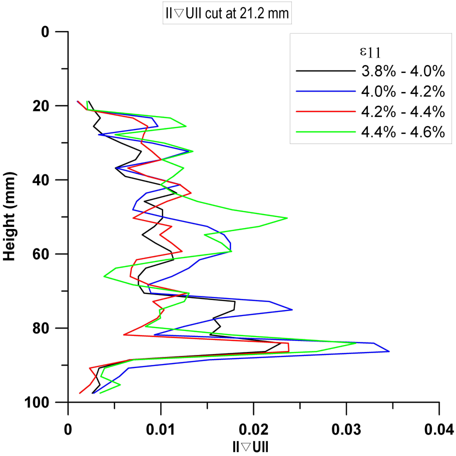
θ^m by Yoshida et al, 1994

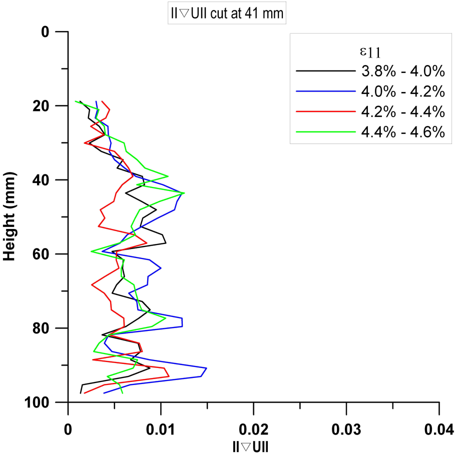
PIV Analysis





PIV Analysis





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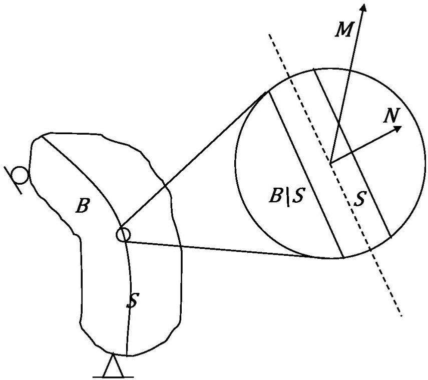


$$\dot{\varepsilon}_S = \dot{\varepsilon}_{B/S} + M \otimes N$$

$\dot{\varepsilon}_S$ and $\dot{\varepsilon}_{B/S}$ are strain rate tensor inside and outside of shear band, respectively.
N Normal unitary vector to shear band.
M is the vector that defines the direction of velocity jump

Applying continuity in the traction vector in the shear band.

$$N \cdot (\dot{\sigma}_{B/S} - \dot{\sigma}_S) = \mathbf{0}$$

Having into account a constitutive equation such as $\dot{\sigma} = \mathbb{C} : \dot{\varepsilon}$

$$(N \cdot \mathbb{C}_S \cdot N) \cdot M = N \cdot (\mathbb{C}_{B/S} - \mathbb{C}_S) : \dot{\varepsilon}_{B/S}$$

$\mathbb{C}_S$ and $\mathbb{C}_{B/S}$ are the constitutive tensor inside and outside of shear band. Working with continuum bifurcation we have

$$(N \cdot \mathbb{C}_S \cdot N) \cdot M = \mathbf{0}$$

Localization condition is satisfaced when the solution is different from trivial one i.e. $(M = \mathbf{0})$

$$\det[N \cdot \mathbb{C}_S \cdot N] = \det[Q(N)] = \mathbf{0}$$

$N \cdot \mathbb{C}_S \cdot N$ is a second order tensor known as acoustic tensor.
In the general case of elastoplasticity, tangent constitutive operator in the case of plastic loading is given by:

$$\mathbb{C}_S = \mathbb{C}^E - \frac{\mathbb{C}^E : \mathbf{m} \otimes \mathbf{n} : \mathbb{C}^E}{\mathcal{H} + \mathbf{n} : \mathbb{C}^E : \mathbf{m}}$$

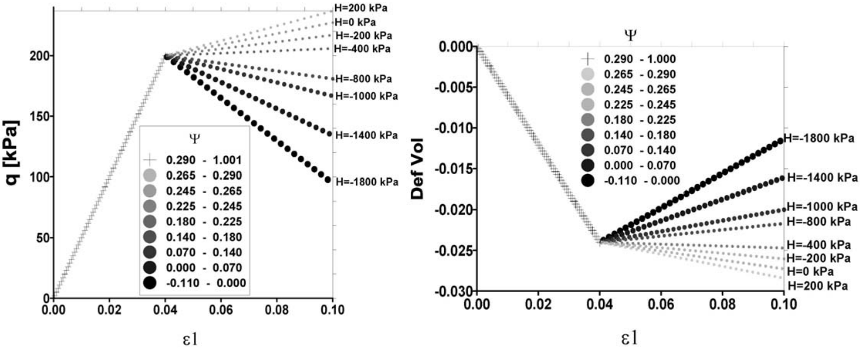
$\mathbb{C}^E$ is the elastic tensor, $\mathcal{H}$ is the plastic module, $n = \frac{\partial F}{\partial \sigma}$ is the normal to yield surface and $m = \frac{\partial G}{\partial \sigma}$ is the normal to plastic potential

$$\mathbb{C}_S = \mathbb{C}^E - \frac{\mathbb{C}^E : \mathbf{m} \otimes \mathbf{n} : \mathbb{C}^E}{\mathcal{K}}$$

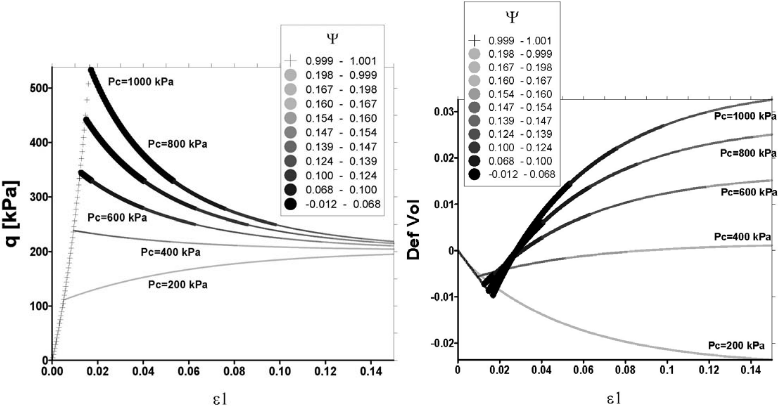
Results of bifurcation analysis in elastoplasticity

It was done to three levels

- Elemental test to prove both performance of constitutive equation and soundness of bifurcation analysis
- Simulation of laboratory test in FEM where it is possible to compare to real ones.
- Numerical modelling of a very simple BVP

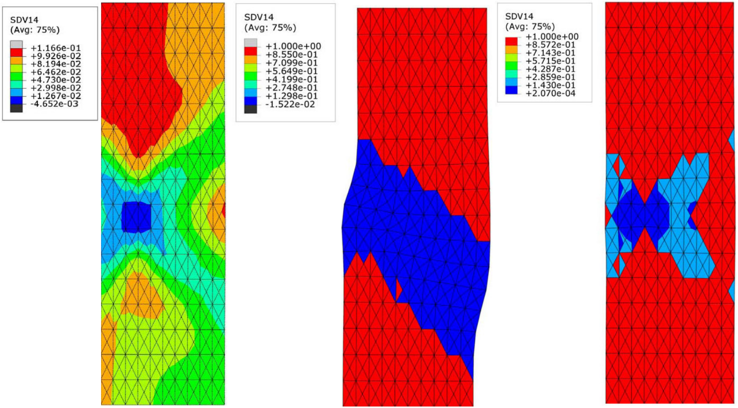


Drained triaxial test using J2 constitutive model



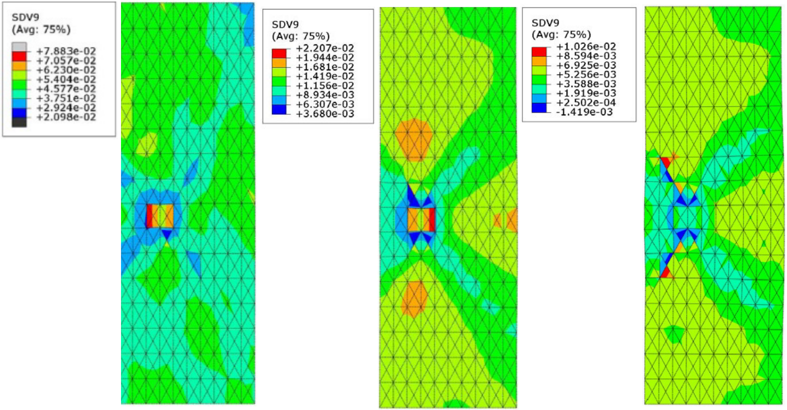
Drained triaxial test using CamClay

Bifurcation applied to biaxial - J2



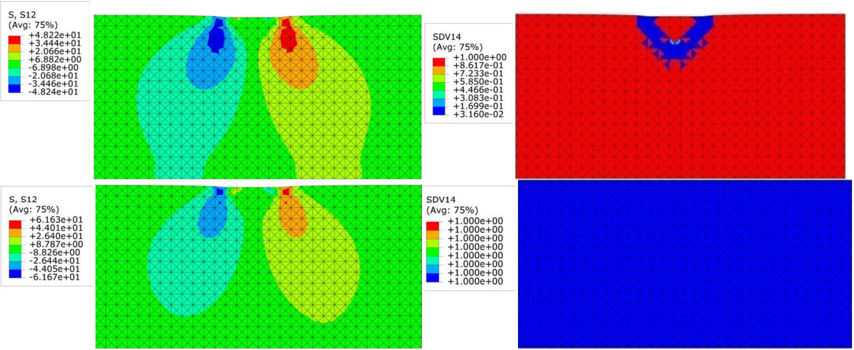
Left $\sigma_Y = 100$ kPa, $\varepsilon_{ax} = 8\,\%$; center $\sigma_Y = 100$ kPa, $\varepsilon_{ax} = 12\,\%$; right $\sigma_Y = 500$ kPa, $\varepsilon_{ax} = 12\,\%$

Bifurcation applied to biaxial - CamClay



$\varepsilon_{ax} = 6\,\%$. Left $p_c = 200$ kPa; center $p_c = 600$ kPa; right $p_c = 1000$ kPa.

Bifurcation applied to BVP. Superficial footing -J2



Vertical displacement 3,33 cm. Top: $\sigma_Y = 100$ kPa, bottom: $\sigma_Y = 200$ kPa.

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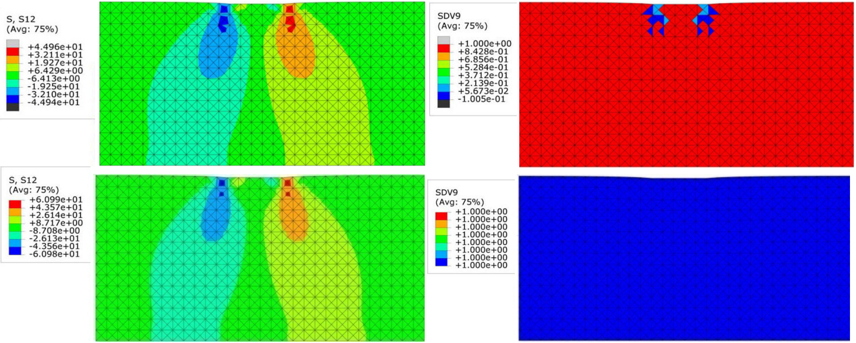
Bifurcation applied to BVP. Superficial footing - CamClay



Vertical displacement 3,33 cm. Top: $p_c = 200$ kPa, bottom: $p_c = 1000$ kPa. Footing using Camclay

Final remarks

- Characteristics of shear bands depends on both constitutive model and meshing used in the FEM analysis.
- The onset of bifurcation depends on elastoplastic tensor. Thus, the modelling of the occurrence or not of the shear band depends on constitutive model.
- It has been developed analytical solutions for bifurcation in special paths. However a more important and useful tool has been implemented in order to be used in FEM.
- Factor Ψ can be used in order to show until what moment the results of simulation of FEM are credible because it does not break any fundamental principles of either continuum media (continuity in the strain field) or calibration of constitutive models (homogeneity in field of stress and strain)

Final remarks

- Possibility of shear band generation is extremely bound with develop of plastic strain in the softening branch. For that reason is more likely shear banding in materials whose both yield stress and softening parameter are low (J2) and highly overconsolidated soils (CamClay).

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