Simulation of an oedometric test using Visco-hypoplasticity

Georgia Institute of Technology, Atlanta
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Geotechnical Research Group
Prof. A. Lizcano
alizcano@uniandes.edu.co

Universidad de los Andes

Bogotá, Colombia

Visco-hypoplasticity in 1D

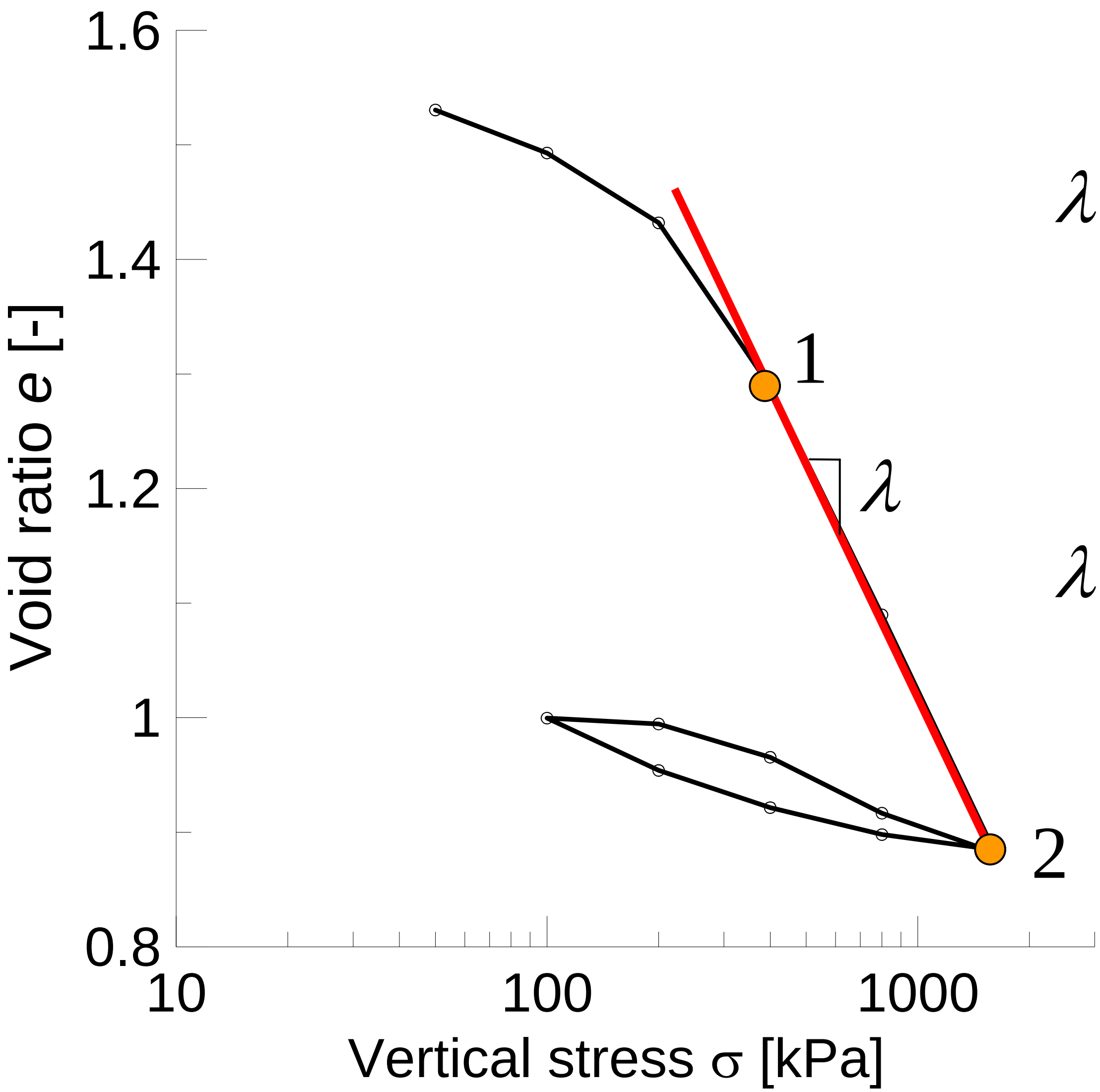
$$\dot{\sigma} = \frac{\sigma\left(1+e_0\right)}{\kappa}\left(\dot{\varepsilon} - \dot{\varepsilon}^{vis}\right)$$

$$\dot{\varepsilon}^{vis} = \dot{\gamma}\left(\frac{\sigma}{\sigma_e}\right)^{\frac{1}{I_v}}$$

$$\dot{\sigma}_e = \frac{\sigma_e\left(1+e_0\right)}{\lambda}\dot{\varepsilon}$$

$\lambda \quad \kappa \quad I_v(\dot{\gamma})$ Material parameters

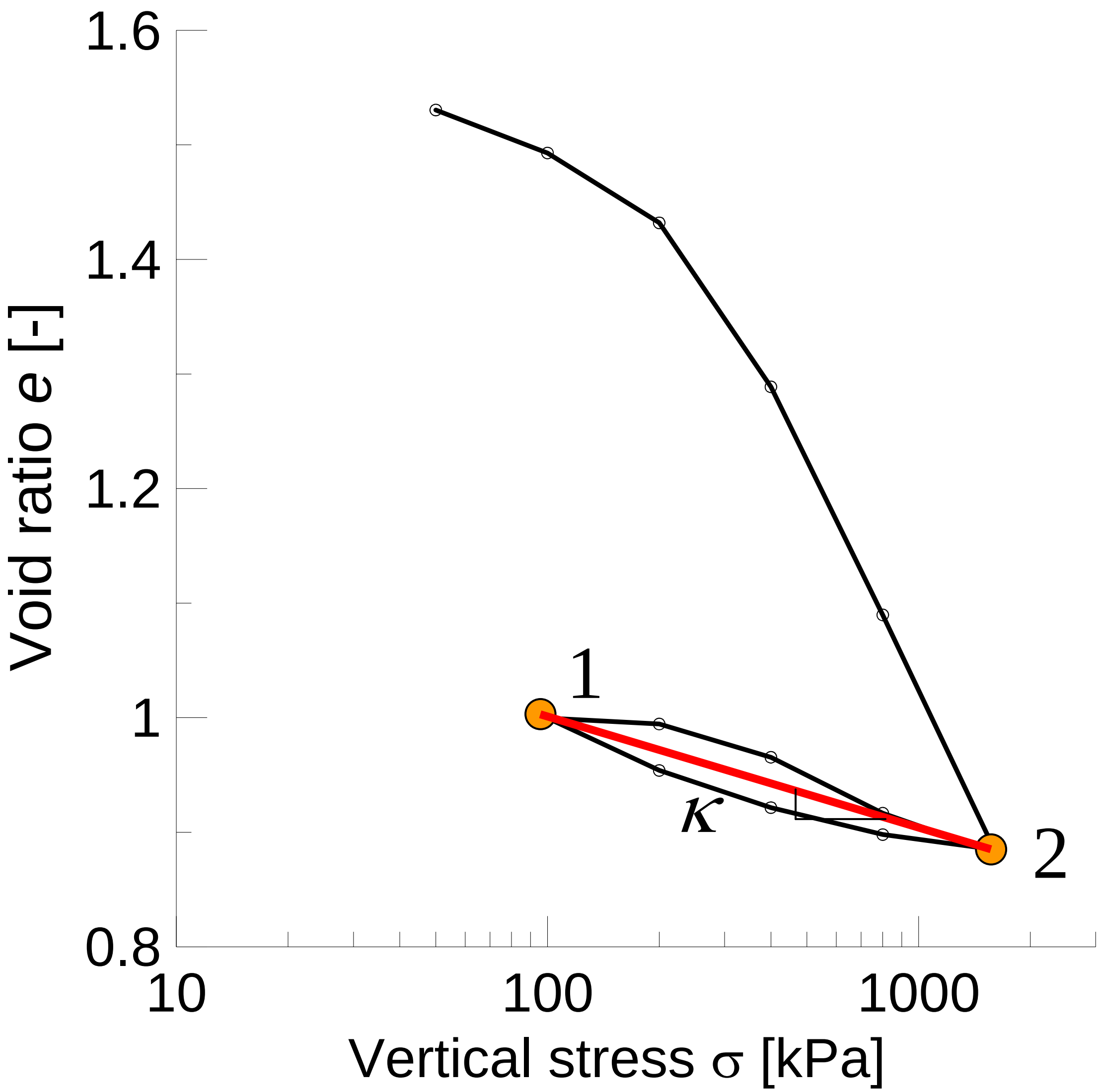
1. Define material parameters



$$\lambda = -\frac{(e_2 - e_1)}{\ln\left(\frac{\sigma_2}{\sigma_1}\right)}$$

$$\lambda = -\frac{(0.89 - 1.29)}{\ln\left(\frac{1600}{400}\right)} = \boxed{0.29}$$

1. Define material parameters



$$\kappa = \frac{(e_2 - e_1)}{\ln\left(\frac{\sigma_2}{\sigma_1}\right)}$$

$$\kappa = -\frac{(0.88 - 1)}{\ln\left(\frac{1600}{100}\right)} = \boxed{0.04}$$

1. Define material parameters

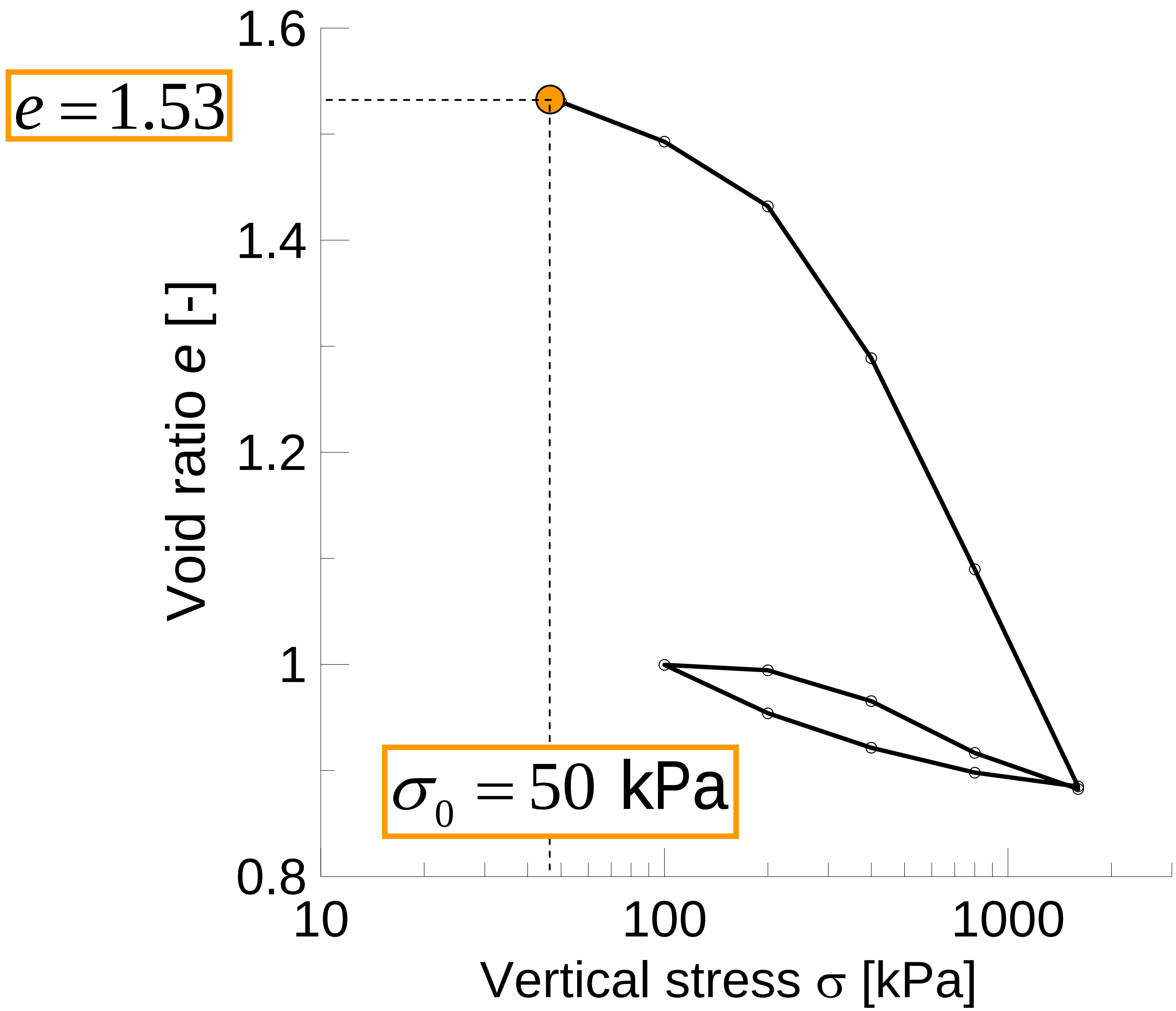
$$I_v = 0.05 + \left(0.026 \cdot \ln\left(w_L\right)\right)$$ Leinenkugel (1979)

$$w_L = 47\%$$

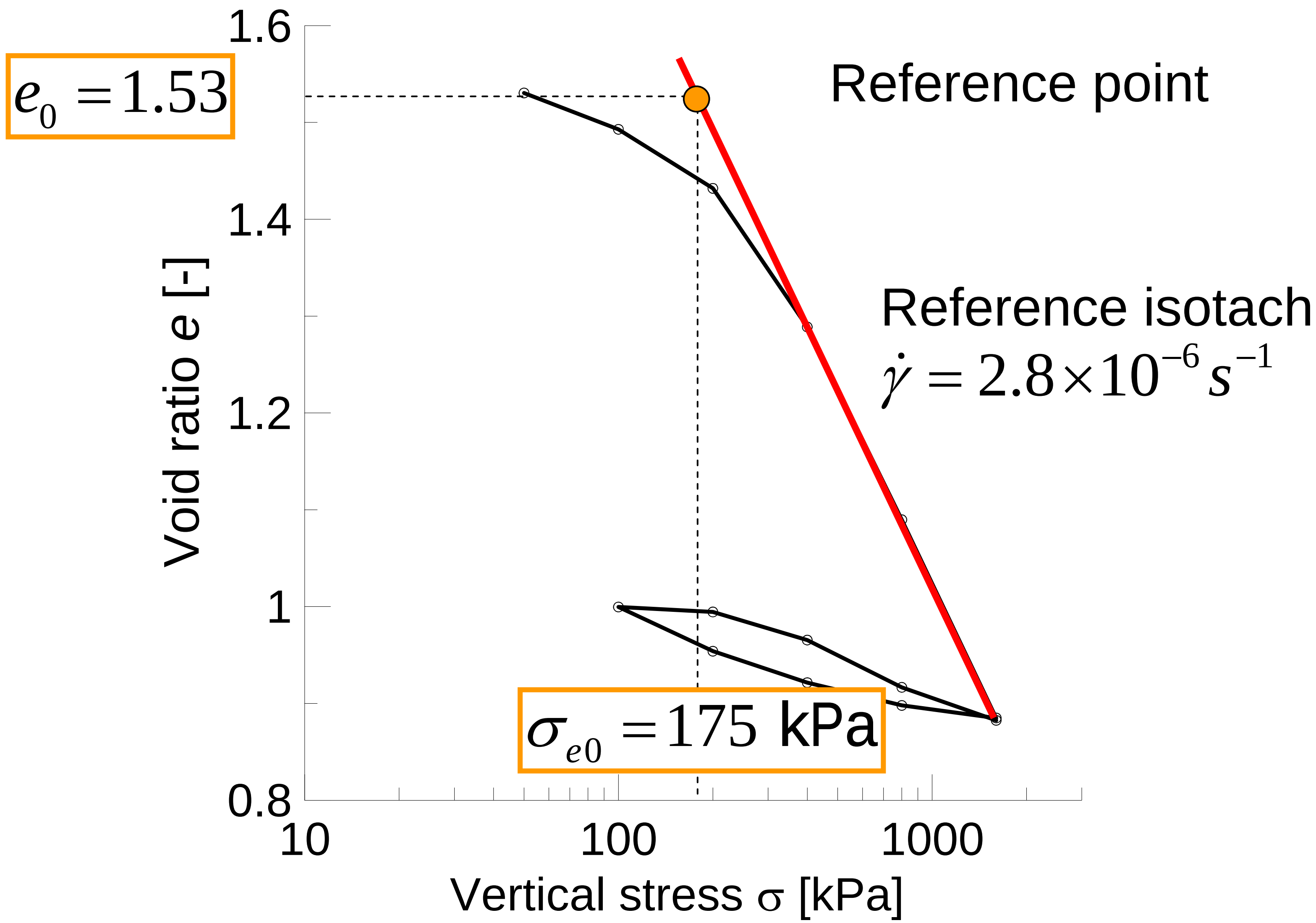
$$I_v = \boxed{0.03}$$

$$\dot{\gamma} = 1\frac{\%}{h} = \boxed{2.8 \times 10^{-6} s^{-1}}$$

2. Define initial stress



3. Define reference values



4. Select a strain rate

$$\dot{\varepsilon} = \dot{\gamma}\left(\frac{\lambda - \kappa}{\lambda}\right) = \boxed{2.39 \times 10^{-6} s^{-1}}$$

5. Define increments
   a. Time increment Δt
   b. Stress or strain increment $\Delta\sigma \; or \; \Delta\varepsilon$
   c. Stiffness $E_i = \dfrac{\sigma_i\left(1 + e_0\right)}{\kappa}$
   d. Calculate

$$\dot{\varepsilon}_i^{vis} = \dot{\gamma}\left(\frac{\sigma_i}{\sigma_{ei}}\right)^{\frac{1}{I_v}}$$

$$\Delta\sigma = E_i\left(\Delta\varepsilon - \dot{\varepsilon}_i^{vis}\Delta t\right)$$

$$\Delta\varepsilon = \frac{\Delta\sigma}{E_i} + \dot{\varepsilon}_i^{vis}\Delta t$$

6. Update state variables

$$\sigma_{i+1} = \sigma_i + \Delta\sigma$$

$$\varepsilon_{i+1} = \varepsilon_i + \Delta\varepsilon$$

$$t_{i+1} = t_i + \Delta t$$

$$\sigma_{e(i+1)} = \sigma_{e(i)} + \frac{\sigma_{e(i)}\left(1+e_0\right)}{\lambda}\Delta\varepsilon$$

Instability

$$\dot{\varepsilon}_i^{vis} = \dot{\gamma}\left(\frac{\sigma_i}{\sigma_{ei}}\right)^{\frac{1}{I_v}}$$

Approximately 20

Function changes rapidly

Calculation becomes unstable

Alternative: to consider the dependence of the viscous rate $\dot{\varepsilon}_i^{vis}$ on the change of strain and stress:

$$a_i = \frac{\partial \dot{\varepsilon}_i^{vis}}{\partial \sigma}$$

$$b_i = \frac{\partial \dot{\varepsilon}_i^{vis}}{\partial \varepsilon}$$

Instability

$$a_i = \frac{\partial \dot{\varepsilon}_i^{vis}}{\partial \sigma} = \frac{\partial}{\partial \sigma}\left(\dot{\gamma}\left[\frac{\sigma_i}{\sigma_{e(i)}}\right]^{\frac{1}{I_v}}\right) = \frac{1}{I_v}\dot{\gamma}\sigma_{e(i)}^{-\frac{1}{I_v}}\sigma_i^{\frac{1}{I_v}-1} = \frac{\dot{\varepsilon}_i^{vis}}{I_v \sigma_i}$$

$$b_i = \frac{\partial \dot{\varepsilon}_i^{vis}}{\partial \varepsilon} = \frac{\partial}{\partial \varepsilon}\left(\dot{\gamma}\left[\frac{\sigma_i}{\sigma_{e(i)}}\right]^{\frac{1}{I_v}}\right) = -\frac{1}{I_v}\dot{\gamma}\sigma^{\frac{1}{I_v}}\sigma_{e(i)}^{-\frac{1}{I_v}}\sigma_{e(i)}^{-1}\frac{\sigma_{e(i)}\left(1+e_0\right)}{\lambda}$$

$$b_i = -\dot{\varepsilon}_i^{vis}\frac{1+e_0}{\lambda I_v}$$

Instability

Including the improvement

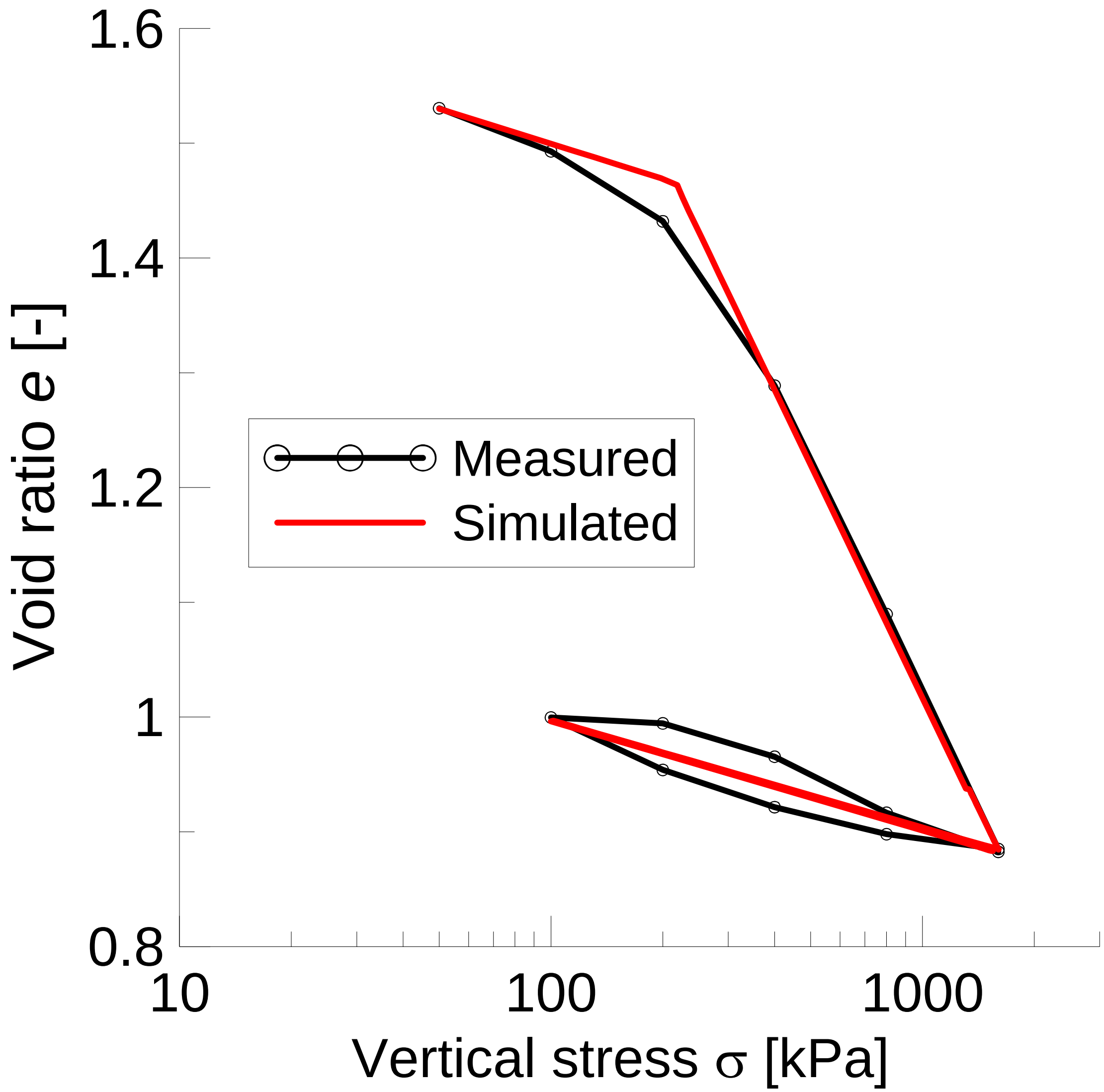
$$\Delta\sigma = E_i\left(\Delta\varepsilon - \left(\dot{\varepsilon}_i^{vis} + \frac{\partial \dot{\varepsilon}_i^{vis}}{\partial \sigma}\Delta\sigma + \frac{\partial \dot{\varepsilon}_i^{vis}}{\partial \varepsilon}\Delta\varepsilon\right)\Delta t\right)$$

$$\Delta\sigma = E_i\left(\Delta\varepsilon - \left(\dot{\varepsilon}_i^{vis} + a_i\Delta\sigma + b_i\Delta\varepsilon\right)\Delta t\right)$$

Possible process to simulate

Process	Input	Value	Output
Load	$\Delta\varepsilon$	$+\Delta\varepsilon$	$\Delta\sigma$
Unload	$\Delta\varepsilon$	$-\Delta\varepsilon$	$\Delta\sigma$
Reload	$\Delta\varepsilon$	$+\Delta\varepsilon$	$\Delta\sigma$
Relaxation	$\Delta\varepsilon$	0	$\Delta\sigma$
Creep	$\Delta\sigma$	0	$\Delta\varepsilon$

Results of the simulation



Excel worksheet

Input data

1. Input values

- Initial conditions:

Initial vertical stress	σ_0
Equivalent vertical stress	σ_e
Initial void ratio	e
Reference void ratio	e_0
Initial strain	ε_0

- Soil parameters:

Slope of compression line	λ
Slope of recompression line	κ
Viscosity index	I_v
Reference rate	$\dot{\gamma}$

Excel worksheet

2. Execution

Execution

Column	Name	Description
A	STEP	0, 1, 2,…
B	dε or dσ	0: If strain increment is given dε 1: If stress increment is given dσ
C	INCREMENT	Increment magnitude. Defined by the user in order to reach the test strain rate
D	Δt	Time increment magnitude. Defined by user
E	TEST STRAIN RATE	$\dot{\varepsilon} = \frac{inc}{\Delta t}$
F	STIFFNESS	$E = \frac{\sigma_{i-1}(1+e_0)}{\kappa}$
G	VISCOUS RATE	$\dot{\varepsilon}^{vis} = \dot{\gamma}\left(\frac{\sigma_{i-1}}{\sigma_{e,i-1}}\right)^{\frac{1}{I_v}}$

Excel worksheet

2. Execution

Execution

Column	Name	Description
H	a	$$a = \frac{\partial \dot{\varepsilon}_i^{vis}}{\partial \sigma} = \frac{\dot{\varepsilon}_i^{vis}}{I_v \sigma_{i-1}}$$
I	b	$$b = \frac{\partial \dot{\varepsilon}_i^{vis}}{\partial \varepsilon} = \frac{-\dot{\varepsilon}_i^{vis}(1+e_0)}{I_v \lambda}$$
J	Δε	If B=0 (dε or dσ $= 0$) $$\Delta\varepsilon_i = inc_i$$ If not $$\Delta\varepsilon_i = \frac{\frac{inc_i(1+E_i a_i \Delta t)}{E_i} + \dot{\varepsilon}_i^{vis}\Delta t}{1 - b_i \Delta t}$$
K	Δσ	If B=1 (dε or dσ $= 1$) $$\Delta\sigma_i = inc_i$$ If not $$\Delta\sigma_i = \frac{E_i}{1+E_i a_i \Delta t} \cdot (1 - b_i \Delta t) inc_i - \dot{\varepsilon}_i^{vis}\Delta t$$

Excel worksheet

2. Execution

Execution

Column	Name	Description
L	STRESS	$\sigma_i = \sigma_{i-1} + \Delta\sigma_i$
M	EQUIVALENT STRESS	$\sigma_e = \sigma_{e,i-1} + \sigma_{e,i-1}(1+e_0)\left(\frac{\Delta\varepsilon_i}{\lambda}\right)$
N	STRAIN	$\varepsilon_i = \varepsilon_{i-1} + \Delta\varepsilon_i$
O	OCR	$OCR = \frac{\sigma_{ei}}{\sigma_i}$
P	TIME	$t_i = t_{i-1} + \Delta t_i$
Q	VOID RATIO	$e_i = e_{i-1} - \Delta\varepsilon_i(1+e_0)$